Leaders, Factions and Party Unity*

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Abstract

We develop a formal model of the intra-party game between the leader and the factions of a mainstream party to study the conditions that are most conducive to the party appearing united behind its leader and electoral program. The leader can be charismatic or not. Charismatic leaders and party unity are both valued by voters. Factions are of interest or of principle. To push factions to contribute to party work and electoral efforts, the leader offers both types of factions their favorite rewards in exchange for their contributions. We show that party unity can only be achieved with charismatic leaders, and it is easier to achieve with factions of interest than with factions of principle. This is especially true if voters observe the factions’ contributions, on top of the rewards they receive. Our results are in line with several marking electoral results of the recent past.

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Abstract

We develop a formal model of the intra-party game between the leader and the factions of a mainstream party to study the conditions that are most conducive to the party appearing united behind its leader and electoral program. The leader can be charismatic or not. Charismatic leaders and party unity are both valued by voters. Factions are of interest or of principle. To push factions to contribute to party work and electoral efforts, the leader offers both types of factions their favorite rewards in exchange for their contributions. We show that party unity can only be achieved with charismatic leaders, and it is easier to achieve with factions of interest than with factions of principle. This is especially true if voters observe the factions’ contributions, on top of the rewards they receive. Our results are in line with several marking electoral results of the recent past.

1 Introduction

For any mainstream political party, a successful electoral strategy requires meeting several conditions. Previous research suggests two of them are of primary importance. First, the party needs to ensure that its leader is valued by voters. Being led by a strong and charismatic leader is thus an advantage, all else equal (Schofield 2003, Bittner 2011, Cross and Pilet 2015). Bill Clinton, Tony Blair and Barack Obama were an asset for their party, John Major, Gordon Brown and Hillary Clinton were not.

Second, the party must appear to voters as being united behind its leader and electoral program. An appealing leader and platform can be quickly undermined if the party displays publicly internal divisions and tensions (Snyder and Ting 2002; Dewan and Myatt 2007, 2008; Andeweg and Thomassen 2011; Ceron 2012; Vivyan and Wagner 2012; Marx and Schumacher 2013; Buttler and Powell 2014; Greene and Haber 2014, 2015). Tony Blair led New Labour to a crushing victory in the 1997 election precisely because he managed to convince his party to support him and the platform he was defending. Blair’s clear overcoming (with a 21 to 3 vote tally in his favor within Labour’s National Executive

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1 Given that our theory rests on the internal ideological heterogeneity of parties, our theory speaks to mainstream parties with a relatively broad appeal, rather than niche and extreme parties, which are typically, though not always, ideologically much narrower and homogeneous.
Committee) of the more ideologically extreme factions within Labour over the need to modify Labour’s Clause IV in March 1995, and thus before his 1997 election landslide victory, is an emphatic case in point. The British newspaper Independent even titled its 14 March 1995 report of this change as “‘Defining moment’ as Blair wins backing for Clause IV” (The Independent, 1995).

For an example from recent American politics, one need not look further than Obama’s 2008 campaign. By generating enthusiasm, goodwill and hope under his “Yes we Can” campaign slogan, Obama’s charisma created precisely such a united front behind him. To the contrary, Hillary Clinton turned out to be quite weak during her 2016 campaign: she did not manage to tame down and control the dissenting voices within her party, especially those coming from the side of Bernie Sanders. Voters were also sceptical about her personality and electoral platform, as she admits herself in her most recent account of her presidential defeat (Clinton, 2017).

But the most recent and outstanding example of the negative consequences of a party being led by a weak leader and not appearing united is the French Socialists’ incredible debacle at the last presidential and legislative elections in May-June 2017. Benoit Hamon, after winning the Socialist party’s primary and becoming the presidential candidate, should have seen all his party rally behind him. Yet, he found himself leading a party in which the different and obviously ideologically diverse factions – like the one supporting Manuel Valls, who after the first round of the Presidential election actually left the Socialist party altogether – were openly supporting and advocating radically different views and policies, such as an alliance with the movement supporting the eventual winner of the presidential race, Emmanuel Macron. The consequences of such a divided front were Hamon’s incapacity to make it to the second round in the presidential election (he actually ended with only 6.36% of the total votes in the first round) and a dismal score – only 31 out of the 577 available legislative seats, the worst score of this party since 1969 – in the legislative elections that followed shortly thereafter. If there are other important forces besides the party’s lack of unity that explain such a dismal result – such as the very poor performance of the previous French President, the socialist François Hollande – Hamon’s weaknesses and in particular his incapacity to seduce the electorate and to hold his
party’s ideological factions firmly behind him clearly did play a major role in this shocking defeat. A final key element of this defeat that speaks to the theory we present below is the fact that the faction which was furthest away ideologically from Hamon, namely that of Manuel Valls, was quite clearly the most vociferous and visible in its attempt to have its views carried by the Socialist candidate. This, as we show below, is exactly what our theory predicts when a party with factions driven by ideology is led by a weak leader.

We develop a formal theory of parties as complex, multi-agent organizations to analyze how the different party factions’ reactions to the position and charisma of the leader impact on how united the party appears in front of the electorate.

Specifically, the problem we analyze is that of a mainstream party – such as the US Republicans or Democrats or the U.K.’s Labour and Conservatives – which has to choose at its annual conference a new leader, knowing that this leader then needs to strike an agreement with its party factions to ensure that both party work and the general election strategy are carried out as well as possible. The analysis focuses on the leader-factions game and we study how its outcome impacts party unity, as a function of the ideological position and types of both the leader and factions.

We consider two main types of factions. They can be either of principle or of interest, to follow the terminology of Bettcher (2005). The first type of faction is essentially ideological in nature and wishes to influence the party manifesto, to reap ‘collective benefits’ from the exchange with the leader, as in the case of Manuel Valls’s faction described above; the second type of faction wishes to obtain party funds or other ‘selective benefits’ that allow it to continue developing and sustaining its factional activities, as was the case for Chicago’s Daley machine (Janda, 1980; Panebianco 1988; Bettcher 2005; Konig 2006; Persico et al. 2011; Dewan and Squintani 2016). A key difference between these two types of factions is that a faction of interest does not care directly about which factions get the funds they did not manage to secure for themselves, as the allocation of these other funds does not impact its activities directly. To the contrary, a faction of principle cares directly about the total distribution of influence across all factions, as this distribution determines the overall electoral message the party sends to voters, and thus can interfere directly with the faction’s own plans. In the terminology of formal modelling, the objective function of
factions of principle exhibits externalities, which are absent from the objective function of factions of interest.

We also consider two types of leader: strong and charismatic or weak and dull. Our definition of charisma is that of *authority charisma*: an individual is authority charismatic when its charisma’s first purpose is to have followers abide by their leadership.\(^2\) Well known examples of authority charismatic leaders, on top of those mentioned earlier, include Baroness Thatcher and General De Gaule. Obviously, voters value charismatic leaders more than dull ones. This is the bright side of having a charismatic leader. Yet, authority charismatic leaders can bias the exchange game they play with factions in favor of those they prefer, as they can always use their authority and electoral appeal as a lever to give their views prominence (Dewan and Myatt 2008; Schumacher, de Vries and Vis, 2013; Landa and Tyson 2017; Bittner 2011). This is the dark side of authority charisma. The main question we ask in the paper is thus: which combination(s) of ideological positions and types for the leader and factions are most likely to allow the party to stand united in front of the electorate?

We model the leader-factions game as an exchange in which the more a faction contributes, the higher is its expected share of influence on the party manifesto – for factions of principle – and of party funds – for factions of interest. The precise exchange technology we employ is the imperfectly discriminating contest success function first developed by Tullock (1980), as it was developed to model games such as the one we consider.

Our modelling of the extent to which the party appears united behind its leader is via both the factions’ contributions and their influence or activities (Miller and Schofield 2003; Schofield and Sened 2005; Schofield 2006). Specifically, the party appears more united the more the available rewards go to the factions that are closest to the leader. That diverse and possibly contradictory influences on the party manifesto can reduce its coherence is self-evident. That the activities of factions of interest also impact the party’s image less so. But they do, as they have a scope, an ideological color that is that of the

\(^2\)Weber (1968, chapter 6) is among the first to discuss this concept. He remarks that this type of charisma can serve the organization – the party, here – only if the leader aligns the effects of their charisma with the objectives of the organization. As in our theory all candidate leaders want (to be chosen by the party, of course, and) want to win the forthcoming election, such a need is automatically satisfied in our case. For a recent popular report on charisma and authority charisma, see BBC (2017).
faction carrying them out — think of the trading of votes for jobs under Daley’s Chicago machine or the public jobs the different factions within Italy’s Christian Democratic party Democrazia Italiana were fighting over (Bettcher 2005; Persico et al. 2011).

We first consider the simplest scenario, in which voters do not observe the factions’ contributions to their party — for example because a substantial fraction of this work is carried out between elections and inside the party, making it hard for voters to observe it — but only their influence, either via the manifesto for factions of principle, or via their factional activities for factions of interest. In this case, we show that a weak leader is never associated to party unity if factions are of principle. To the contrary, with factions of interest, party unity is easier to achieve: a weak leader delivers some unity and a strong leader is always associated with party unity, no matter what ideological position he has.\(^3\)

The reason why it is difficult to achieve a united front when factions are of principle is that, because factions of principle care directly about the influence other factions have on the party manifesto, the faction(s) closest to the leader typically have an incentive to contribute less and free ride on the countervailing influence attempts of the other, more distant factions. But then the party cannot show an effective united front to voters, as the party finds itself with a manifesto that reflects the views of actors (the leader and the factions most distant from them) that are unlikely to be compatible, let alone consistent with each other. This is actually always true when the party leader turns out to be weak, consistently with the French elections example above.

Extending our game to the case in which voters also observe the factions’ contributions reinforces our previous findings as, when factions are of principle, it becomes simply impossible to find a combination of ideological locations for the leader and the factions such that the party appears united behind its leader — weak or strong. To the contrary, this is always possible with factions of interest, and also especially easy when the leader’s location is relatively close to their party’s ideological center.

\(^3\)One provocative way to interpret the finding that party unity requires a strong leader who can bias the exchange with factions is that too much internal democracy, if it prevents strong leaders from biasing the internal workings of the party their way, leads to potential cacophony and visible tensions between the party’s different factions, thus reducing the electoral appeal of the party. Of course, internal democracy has many merits too; our results only highlight a potential trade off inherent in the internal democracy of parties.
The general conclusion of our analysis is thus that factions of interest should be more easily associated to party unity than factions of principle, all else equal. More precisely, the set of ideological location of the leader that are consistent with strong party unity is always larger with factions of interest than with factions of principle. On top of the examples we gave above, many of the accounts in Janda (1980), Bettcher (2005), Persico et al. (2011) and Ceron (2012) also point to the importance of party unity for electoral success. For example, Ceron (2012) shows empirically that factions within Italian parties between 1946 and 2010 constrained the position of the leader because of the need for party unity. Our formal model puts forward a mechanism that suggests that the constraints on party unity are tighter with factions of principle.

The rest of this paper is structured as follows. We first discuss some important related literature. Section 3 introduces the model and the next section solves for the equilibrium of the game when factions are of interest. Section 5 solves for the equilibrium of the game when factions are of principle. The following section extends our game to the case in which voters observe the factions’ contributions. The last section concludes and offers avenues for further research.

2 Related Literature

The main goal of our analysis is to study how different types of leaders impact on the willingness of different types of factions to contribute to their party’s work and electoral strategy and thereby allow the party to stand in front of the electorate with a more or less united front behind the leader and the electoral platform.

We build on three strands of the existing literature. First, we borrow from the literature on both valence politics and the personalization of politics the idea that strong and charismatic leaders bring about an electoral bonus, all else equal. Important contributions are Schofield (2003), Bittner (2011) and Cross and Pilet (2015). For example, Schofield (2003) builds a formal model that adds to the literature on spatial electoral competition by letting the party leader be of high or low valence, and Bittner (2011) shows empirically that the charisma of leaders – as opposed to their competence, for example – is
the most important factor determining success in elections. Very recently, Wauters et al. (Forthcoming) also show that, at least in the Belgian flexible proportional system, the personalization of politics seems to be increasingly limited to party leaders, in line thus with our working assumption that having a strong and charismatic leader is an electoral advantage, all else equal. We add to this literature by suggesting one additional consequence of having a strong and charismatic leader that may be important for the electoral score of a party: the degree of internal unity a party achieves with such leaders.

Second, we also take seriously the idea of the literature on valence politics that a party’s main activists and factions are key to determine the party’s electoral fortunes. See for example Miller and Schofield (2003), Schofield (2006) and the literature that builds on these articles, such as Schofield and Sened (2005); Karp, Banducci, and Bowler (2007); Schofield et al. (2011) and Kernell (2015). Our contribution here is to allow factions to be of one of the two main types – namely of interest or of principle, following Bettcher (2005) and Persico et al. (2011) – to study the willingness of different types of factions to contribute under different types of leaders.

Third, our formal theory contributes to the literature on the electoral consequences of party unity by suggesting one mechanism through which the interactions between the position and type of the leader and that of the factions map into the degree of party unity that can be achieved. Caillaud and Tirole (1999, 2002) were among the first to propose a formal model in which disagreement between factions can reduce the electoral appeal of the party, but their theory offers no explicit role for the party leader and their views. Other formal theories which focus on the effects of disagreements between factions are Hortala-Vallve and Mueller (2015) and Mutlu-Eren (2015). In these contribution it is the effect of party splits that takes center stage. Dewan and Myatt (2007 and 2008) are contributions which focus on the role and effects of the characteristics and choices of a party leader, but in their theory factions play not explicit role. Empirical exercises in this strand of the literature include Andeweg and Thomassen (2011), Ceron (2012), Vivyan and Wagner (2012), Marx and Schumacher (2013), Buttler and Powell (2014) and Greene

\footnote{Aldrich (1983) is one of the very first formal models that includes activists, yet in that article activists do not influence the platform of the party.}
and Haber (2014 and 2015). Greene and Haber (2015) are especially complementary to our contribution as they show empirically that party disunity is viewed negatively by voters, all else equal.

3 The Model

The problem we analyze is that of a mainstream party which has to choose its new leader at its annual conference, keeping in mind that it must maximize its chances of winning the upcoming general election. Suppose without loss of generality that the ideological spectrum of the party is \([0, 1]\). The set of candidate leaders also spans the party’s ideological spectrum. Each leader is characterised by an ideological bliss point.\(^6\) The chosen leader’s ideological location \(L\) is also that of the party manifesto.

At the time of the selection of the leader and the subsequent exchanges between the leader and the factions, it is common knowledge that any party has the same ex-ante probability of winning the election, irrespective of the ideological color of their leader and thus the ideological color of the manifesto.\(^7\) The uncertainty surrounding the location of the median voter allows us to focus on the intra-party game between the leader and factions.

In line with the literature on valence politics – see for example Schofield (2006) – the electoral environment is such that the leader and factions need to work together to maximize the party’s chances of winning the election. We denote by \(v_L\) the leader’s authority charisma, their capacity to have followers or subordinates obey their command. The leader’s authority charisma can be weak \((w)\) or strong \((s)\): \(v_L = \{w, s\}\). The party

\(^5\)Of course, party unity is influenced by other factors besides the type and positions of factions and the party leader. For example, the electoral system and the candidate selection procedure have been shown to matter. Carey and Shugart (1995) is the classical reference on the influence of electoral systems. For a recent empirical exercise on the role of the candidate selection procedure, see Shomer (forthcoming). To maintain expositional clarity, our theory focuses only on the effects of the leader-factions relationship.

\(^6\)We could also assume explicitly that all candidate leaders care about winning the election, but such an assumption is not necessary in our theory, as the analysis below will show.

\(^7\)One avenue to model this is simply to assume that: 1) parties know that the median voter’s bliss ideological point is a random variable whose realization will be drawn just before the election; and 2) all parties currently know is that this will be a draw from the Uniform distribution on the full, election-wide ideologic spectrum. The analysis of this extended game is straightforward using any standard probabilistic voting game and is available upon request.
competes in an electoral system where an authority charismatic leader is an electoral advantage, all else equal, for example because voters value inherently such a leadership trait.\textsuperscript{8} Thus, whenever $v_L = s$, the party receives an electoral boost. This is the bright side of charisma. Yet, generally, any leader, like any person, wishes to give more prominence to the views that are more in line with theirs.\textsuperscript{9} An authority charismatic leader thus also comes with a dark side: his authority charisma puts him in a position to bias any intra-party decision in favor of the actors that are closest to his views. A weak leader cannot meet this desire of his.

Factions are modelled as either factions of principle, which are populated by politicians motivated by ideology, or factions of interest, which are populated by politicians motivated by the desire to obtain personal benefits, which we model hereafter as monetary funds. For the sake of simplicity, we assume that there are three equally-sized factions of each type, labelled $A$, $B$ and $C$ – as will soon become clear, we do not need to have separate labels for factions of interest and principle. Each faction is characterized by an ideologic position on its party-specific subset of the overall ideological spectrum. For either factional type, $A$ has its ideologic bliss point in 0; $C$ has it in 1 and that of $B$ is somewhere in $[0, 1]$\textsuperscript{10}.

Each faction of principle would like to craft the party manifesto in line with its ideologic preferences, which may not be in line with those of the leader, of course. Factions of interest would like to get funds from their party to be able to carry put their factional activities, but the ideological color of such activities may again not be aligned with those of the leader. And no factions will offer the leader any help for free. If there can be diverse reasons why factions may need to be compensated to follow their leader’s desires and choices, our way of modelling this aspect of the problem is simply by making contributions to the party costly for factions. The leader thus sets up two exchanges with factions, one per faction type. In the exchange with factions of interest, the leader offers party funds worth $V$ in exchange for each faction’s contribution. These funds can then be used by

\textsuperscript{8}All the electoral systems that are in Bittner’s (2011) analysis appear to have such a characteristic. Thus, this is a fairly ubiquitous aspect of elections.

\textsuperscript{9}See for example Schelling (1978) and Benhabib, Bisin and Jackson (2011) on the pervasiveness of such homophily in very diverse environments.

\textsuperscript{10}The reason why factions of interest have an ideological color too is simply that the actions they undertake are obviously in line with their views. The examples we reported in the Introduction clearly demonstrate this.
the different factions to fund their own activities. Obviously, such activities will have the faction’s preferred ideological color, which may not necessarily be that of the leader. As the use of such funds is obviously observable to voters, the ideological diversity of the activities financed by these funds generate in the electorate a perception that the party is not united behind the leader. In the second exchange, with factions of principle, the reward offered to factions in return for their contributions is influence on a share \( I \) of the party manifesto.\(^{11}\) Here too, the factions’ influence on the different parts of the party manifesto impacts the homogeneity, the unity of the ideological message the leader and the party send to voters at election time. Of course, the actual contributions of the factions could also impact the voters’ view of the degree to which the party is united.

In the main body of the paper, we consider these contributions to be unobservable and payoff-irrelevant to voters — these represent thus any ‘behind closed door’ activity that needs to be carried out for the party to survive between (but also during) elections, for example. In Section 5 we extend our game to the case in which party work is observable and payoff-relevant to voters.

Let \( c_i \) be faction \( i \)’s contribution to the party. In the spirit of Tullock (1980), the mapping governing the exchange is given by the following standard imperfectly discriminating contest success function:

\[
Sh_i = \frac{b_i c_i}{\sum_{j=A,B,C} b_j c_j}
\]

where \( Sh_i \) is faction \( i \)’s share of party funds or party manifesto influence and where the different \( b_i \)'s are the weights the leader uses in evaluating each faction’s contribution to party work. All weights are between zero and one. The closer to one is \( b_i \), the more the leader favors faction \( i \). The leader’s capacity to bias the exchange with factions is the second facet of the leader’s strength. If the leader is weak, the exchange with factions is unbiased and \( b_i = 1 \) for all \( i = A, B, C \). If the leader is strong, the exchange is biased and the biases the leader uses are given by one minus the Euclidean distance between the

\(^{11}\) We split the exchange between the leader and the factions into two different contests for the sake of expositional ease. Indeed, as the goals of the two types of factions differ and so do the rewards available, the choices of one set of factions does not influence directly the choices of the other set of factions. Thus there is not too much loss of generality in splitting the exchange the way we do.
leader’s ideological position $L$ and that of any faction $i$. Let $L \in [0, 1/2]$ (the biases are defined symmetrically for $L \in (1/2, 1]$), then the biases of the strong leader are given by:

- If $B \geq L$:

  \[ b_A = A - L = 1 - L \]
  \[ b_B = 1 - B + L \]
  \[ b_C = L - C = L \]

- If $B < L$:

  \[ b_A = A - L = 1 - L \]
  \[ b_B = 1 - L + B \]
  \[ b_C = L - C = L \]

In the exchange with factions of interest, faction $i = A, B, C$ solves the following problem:

\[
\max_{c_i} \frac{b_i c_i}{\sum_{j=A,B,C} b_j c_j} V - c_i
\] (1)

where the second, negative term in the objective function of faction $i$ stands for the cost of supporting the party and its leader. We also make the conservative assumption that the leader’s type does not influence $V$.$^{12}$

Turning to the exchange with factions of principle, we do not take a strong stance on how leadership strength impacts the share of the manifesto that is open to influence: $I$ is independent of the leader’s type.$^{13}$ Contrary to factions of interest, factions of principle,

\[^{12}\text{In line with the available empirical evidence, see for instance IDEA (2013), we view these funds as being determined mainly by the party’s performance at the previous election, and thus factions realize that their choices regarding their current contribution to party work cannot influence the value of } V, \text{ which is predetermined at the moment they make their choice. Assuming that the amount of available party funds under a strong leader is at least as large as that available under a weak leader reinforces all the results that follow.}\]

\[^{13}\text{Assuming that a weak leader opens to influence at least as large a share of the manifesto as a strong leader, which would seem the obvious assumption to make in case we wish } I \text{ to depend on the type of leader in charge, reinforces all the results that follow.}\]
as they care about the overall ideological stance of the party, also care directly about how the other factions of principle influence the party manifesto. And, logically, any faction of principle would rather see other factions who are ideologically close to them influence the manifesto. Thus, for ideological factions, the objective function takes into account the influence of other factions, and the weight attached to each of the other factions’ influence is decreasing in the (Euclidean) ideological distance between them and these other factions. There are thus externalities in their objective function, to borrow from the terminology of formal modelling.

Faction \( i = A, B, C \) solves the following problem:

\[
\max_{c_i} \frac{b_ic_i}{\sum_m b_m c_m} I + (1 - d_{ij}) \frac{b_j c_j}{\sum_m b_m c_m} I + (1 - d_{ik}) \frac{b_k c_k}{\sum_m b_m c_m} I - c_i
\]

where \( d_{AC} = A - C = 1 \), \( d_{AB} = B - 0 = B \), \( d_{BC} = C - B = 1 - B \).

Voters care about three aspects of the platform proposed by the party: 1) the ideology defended by the leader of each party; 2) the leader being strong and charismatic; and 3) the party appearing as united and coherent as possible behind the leader. As the party is in the dark about the position of the median voter at the time of choosing its leader, the party will thus focus on choosing a leader who delivers party unity, if such a leader is available, of course. And if both weak and charismatic leaders are equally ‘useful’ at delivering unity, it will select a charismatic one.

Denote party unity with \( U \). For either factional type, we shall say that strong party unity is achieved, \( U = S \), if the mapping from the factions’ distance from the leader to party funds share or manifesto influence is strictly decreasing: the more distant a faction is, the less funds/influence it gets. Indeed, with such a decreasing mapping, the message the party and the leader send to voters is most coherent. If the mapping from distance to funds/influence is decreasing but not strictly so, we have weak party unity, \( U = W \). If the mapping is not monotonically decreasing in distance, we have no party unity, \( U = 0 \). For the sake of simplicity, we operationalize the electoral effect of our definition of party unity above into a discrete payoff for the party, \( V(U) \). This payoff takes on value \( V(S) = 1 \) whenever the mapping from distance from the leader to party
funds share or manifesto influence is strictly decreasing; \( V(W) = 1/2 \) if the mapping from distance to funds/influence is decreasing but not strictly so; and \( V(0) = 0 \) if the mapping is not monotonically decreasing in distance.\(^{14}\)

Let \( \mu(v_L) \) be the party’s direct electoral payoff associated to the charisma of the leader, with \( \mu(s) > \mu(w) = 0.\(^{15}\) Given the party’s uncertainty about the location of the median voter at election time, the objective of the party is thus to maximize \( \mu(v_L) + V(U) \).

Given that, in equilibrium, we will never have to compare cases in which the leader is weak and strong party unity is achieved with cases in which the leader is strong but party unity fails completely, we do not have to take a stand on the ranking between \( \mu(s) + V(0) = \mu(s) \) and \( \mu(w) + V(S) = 1 \).

The timing of the game is as follows:

1. The party selects its leader
2. The three factions of each type exert effort
3. The three factions receive their reward, which define whether or not the party achieves unity
4. Nature sets the ideological location of the median voter
5. The median voter observes, for each party, the location of the party manifesto, the leader type, whether or not the party is united and cast their ballot for their favorite party.

As we do not solve explicitly for the party’s decision at stage 1 – we do not need to do so explicitly to pin down the combinations of leader-faction ideologies that the party prefers – and we focus on the factions’ decision at stage 2, our solution concept is Nash equilibrium.

\(^{14}\)Using a continuous measure such as \( \sum_i (1-d_{Li})S_i k_i^* \) yields the same results as the mapping from distance to funds/influence for the factions that participate in the exchange turns out to be of three types only: 1) strictly decreasing, 2) weakly decreasing; or 3) neither of the previous two cases.

\(^{15}\)The fact that \( \mu(w) = 0 \) is just a normalization.
4 Equilibrium Factional Behavior

4.1 Factions of interest

Suppose first that the party could not select a strong and charismatic leader. The leader is thus weak and the exchange procedure for the allocation of the party funds is unbiased: all factions compete for the funds on an equal footing. In such a case, the share of the funds going to faction $i$ is exactly proportional to their contribution to the party, $c_i$:

$$Sh_i = \frac{c_i}{\sum_j c_j}$$

Then, equilibrium behavior is symmetric, as each of them contributes equally and receives the same share of party funds:

**Proposition 1.** Suppose the leader is weak so that the exchange with factions of interest is unbiased. Then, for all $i = A, B, C$, we have:

- $c^*_i = \frac{2V}{9}$;
- $Sh^*_i = \frac{1}{3}$;
- Party unity is weak.

**Proof.** The proof of Proposition 1 and all other results that follow are relegated to the Appendix.

The exchange between a weak leader and factions of interest reduces to a standard Tullock (1980) contest with identical players, in which the equilibrium contribution is increasing in the value of the available reward and decreasing in the number of participants in the contest. As the share of the party funds is uniform across participating factions, the party achieves weak unity under a weak leader. Its payoff is thus $\mu(w) + V(W) = 1/2$.

Can the party deliver a more focused message and image when the leader is authority charismatic? With such a leader, the exchange procedure is biased towards those factions who are closer to the leader, and equilibrium behavior across factions is more involved. First of all, when one of the factions is too far from both the leader and the other two
factions, this isolated faction decides not to contribute. The intuition for why this isolated player decides to drop out of the exchange is quite simply that they feel that the procedure is so biased against them that there is no point in participating in it. Further, whenever one faction does not participate in the exchange with the leader, the two active factions behave symmetrically, even though they are subject to different biases in the exchange procedure. This implies in turn that the active faction which obtains the largest share of the party funds is the one with the bliss ideological point closest to that of the leader. We thus have:

**Proposition 2.** Suppose the leader is authority charismatic. In any equilibrium in which only factions of interest $i$ and $j$ contribute, we have:

- $c_i^* = c_j^* = \frac{b_i b_j}{(b_i + b_j)^2} V$;
- $Sh_i^* > Sh_j^*$ if and only if $d_{Li} < d_{Lj}$;
- Party unity is strong.

The payoff of the party in this case is: $\mu(s) + V(S) = \mu(s) + 1$.

What if the positions of the three factions and the leader are such that all three factions actively participate in the exchange with the leader? Then we have the following equilibrium contributions and party fund shares:\(^{16}\)

**Proposition 3.** Suppose the leader is authority charismatic. When all three factions of interest participate in equilibrium, we have:

1. If $B$ is closer to $L$ than $A$ (resp. $C$), then:
   (a) $B$ contributes most, followed by $A$ and then $C$ (resp. $C$ then $A$);
   (b) $B$ also receives the largest share of party funds, followed again by $A$ and then $C$ (resp. $C$ then $A$).

2. If $B$ is further away from $L$ than $A$ (resp. $C$), then:

\(^{16}\)In the proof of this proposition in the Appendix, we pin down precisely all parameter configurations leading to all three factions being active.
(a) $A$ (resp. $C$) contributes most, followed by $B$ and then $C$ (resp. $A$);
(b) $A$ (resp. $C$) also receives the largest share of party funds, followed again by $B$ and then $C$ (resp. $A$).

3. Party unity is always strong.

In this case too, the payoff of the party is: $\mu(s)+V(S) = \mu(s) + 1$. Thus with factions of interest, the party is always better off with a charismatic leader: such a leader delivers not only a direct electoral boost, through the charisma of the leader, but also allows the party to achieve strong party unity, which again boosts its electoral fortunes.

Figure 1 is a stylized representation of all the equilibria of the exchange between an authority charismatic leader and factions of interest, as a function of the ideological position of $B$ and $L$. The figure also reports the ranking of factional contributions and fund shares.

![Figure 1: Contributions and fund shares of factions of interest](image)
Let us focus on the left-hand half of the figure, for the right-hand side of the figure the reasoning is symmetric. An intuition for the shapes and locations of the areas where the faction furthest away from \( L \), is inactive is as follows. Imagine that \( L \) is close to \( A \), in 0.2, say, and so is \( B \), in 0.3 say. Then \( C \) has a very low probability of receiving any funds because of his very low bias and the large biases that both \( A \) and \( B \) enjoy. As any contribution is costly to \( C \), \( C \) is better off dropping out of the exchange. Now move \( B \) closer to \( C \) and further away from \( L \), keeping all else equal. Then the share of funds \( C \) can hope to obtain increases because the leader favors \( B \) less and \( C \) (and \( A \)) relatively more (the bias of \( B \) decreases hence the expected share of funds of \( C \) increases because of the decrease in the denominator of \( Sh_C \)). Hence, as \( B \) gets closer to \( C \), there can be a position of \( B \) beyond which \( C \) starts to participate in the exchange.

Let us now turn to the intuition for why the the set in which only two factions are active has a missile-like shape. Focus on the area in the figure with \( L \) around 0.35. Start from a point where all three factions are active, say at \( B \) in 0.9. Then, as \( B \) gets closer and closer to \( L \), \( B \)’s expected share of party funds increases, to reach a pinnacle for \( B = L \). This must depress (\( A \)’s and) \( C \)’s incentives to participate in the exchange (or more generally the incentives of the faction who is furthest from \( L \)) which implies that there is a set of combinations of ideological positions of \( B \) and \( L \) such that \( C \) does not participate.

### 4.2 Factions of Principle

How do factions of principle behave when their leader is weak and thus the exchange is unbiased? As with factions of interest, all factions are then on an equal footing. Yet, these factions also care directly about how the other factions influence the manifesto, so we should expect factions’ behavior not to be symmetric even when the exchange is unbiased. Indeed, when the leader is weak and thus the exchange is unbiased, we have:

**Proposition 4.** Suppose the leader is weak so that the exchange with factions of principle is unbiased. Then, in equilibrium we have:

- \( c_A^* = c_C^* = \frac{1}{4} \) and \( c_B^* = 0 \);
- \( Sh_A^* = Sh_C^* = \frac{1}{2} \) and \( Sh_B^* = 0 \);
• There is no party unity.

With factions of principle, when the leader is weak, only two factions, $A$ and $C$, actively participate in the exchange: $B$ does not find it worthwhile to participate, the two active factions behave symmetrically and are offered the same influence on the manifesto.\textsuperscript{17} Thus, with a weak leader and factions of principle, the mapping from distance to the leader and influence is not monotonic: party unity is absent. The message to the electorate that comes out of the party manifesto is likely to be highly unclear and unappealing to voters, as the views that are contained in there are those of the two most distant factions and thus are somewhat incoherent, if not altogether incompatible. The party’s payoff in this case is a very dismal 0.

The behavior of the different factions of principle within the French Socialist party at the last presidential and legislative elections of 2017 are very much in line with this prediction: Benoit Hamon, the socialist leader, chose a markedly leftwing platform but turned out to be a very weak and electorally unappealing candidate. As predicted by our model, the faction of Manuel Valls, which was the one furthest away from that of the Socialist leader, was also the most active and vociferous in the election. The party thus appeared deeply divided. And the end result was an incredibly dismal score in both the presidential and legislative elections.

Suppose now that the leader is authority charismatic so that the exchange is biased towards those factions which are ideologically closer to the leader. As was the case for factions of interest, whenever one faction does not participate in party work in equilibrium, the two active factions behave symmetrically even though they are subject to different biases in the exchange procedure. Hence, the faction which ends up being most influential over the party manifesto is the one whose bliss ideological point is closest to the one of the leader and party unity is thus strong. We have:

**Proposition 5.** Suppose the leader is authority charismatic. In any equilibrium in which

\textsuperscript{17}This result is reminiscent of similar findings in the economics corporate governance literature. See for example, Osborne et al. (2000) and Flamand and Troumpounis (2014), who provide two different but related formal models in which moderate individuals – like believer $B$ in our model – tend to not participate in costly meetings when participants are principled instead of interested, to use our factional terminology.
only two factions of principle $i$ and $j$ participate in the exchange with the leader, these two factions are the ones that are closest to the leader, and:

- $c_i^* = c_j^* = \frac{b_i d_{ij}}{(b_i + b_j)} I$;

- $Sh_i^* > Sh_j^*$ if and only if $d_{L_i} < d_{L_j}$;

- Party unity is strong.

The party payoff in this case is, like with factions of interest, maximal and equal to $\mu (s) + 1$.

What does participation look like when all three factions of principle are active? We have:

**Proposition 6.** Suppose the leader is authority charismatic. When all three factions of principle contribute, which can happen only if $B$ is closer to $L$ than $A$ (resp. $C$), we have:

1. If $A$ (resp. $C$) and $B$ are on opposite ideologic sides of $L$, then:
   
   (a) $A$ (resp. $C$) contributes most, followed by $B$ and then $C$ (resp. $A$);
   
   (b) $B$ has the largest influence on the manifesto, followed by $A$ (or $C$) and then $C$ (or $A$).

2. If $A$ (resp. $C$) and $B$ are on the same ideologic side of $L$, then:

   (a) $C$ (or $A$) contributes most, followed by $B$ and then $A$ (resp. $C$);
   
   (b) $B$ has the largest influence on the party manifesto, followed by $A$ or $C$.

3. Party unity is strong if and only if $C$ (resp. $A$) has the least equilibrium influence on the party manifesto; otherwise there is no party unity.

Thus, depending on the ideology of the authority charismatic leader, the party’s payoff is either $\mu (s) + 1$ or $\mu (s)$.

Figure 3 displays the set of equilibria with factions of principle. The hashed area is the area in which there is no party unity.
Figure 2: Contributions and influence shares of factions of principle

An intuition for the shapes and locations of the areas where $C$, the faction furthest away from $L$, is inactive is as follows. Imagine that $L$ is close to $A$, say at 0.2. If $B$ is very close to $C$, say at 0.9, it means that $C$ has high incentives to free-ride on $B$, as their ideology is very close, and $A$ and $B$ have low incentives to free-ride on each other, hence their contributions are rather large. Thus, it is not worthwhile for $C$ to contribute to the party cause. As $B$ moves closer to $A$, $C$ has lower incentives to free-ride on $B$, whereas $A$ and $B$ have higher incentives to free-ride on each other, meaning that the contribution they provide decreases. The fact that the bias of $B$ increases reinforces this process, as it allows $B$ to reduce its contribution while keeping the same expected influence. After some point (namely when $B < 2L$), $C$ finds it worthwhile to devote some time and energy to the party cause. This mechanism goes on as $B$ moves even closer to $A$ until reaching the position of $L$, at which point the three factions contribute equally. Finally, when $B < L$, and thus $A$ and $B$ are very close to each other, it is $C$ who contributes the most: it has to compensate for the large biases in favor of $A$ and $B$, whose ideology it dislikes a lot.

Further, notice that the factions’ location configurations such that $C$ is inactive are very different from the ones relative to factions of interest. With factions of interest, $C$ does not contribute when $L$ is too far from them, unless $B$ is quite close to $C$ so that $C$ does not feel isolated ideologically. To the contrary, with factions of principle, $C$ does not
contribute if \((L \text{ is too far from } C \text{ and } B \text{ is quite close to } C)\), then \(C\) can free-ride on \(B\)'s contribution to see the manifesto being influenced along lines that \(C\) does not dislike too much.

The overall conclusion of the analysis in this section is that factions of interest are more conducive to party unity than factions of principle, regardless of the charisma of the leader. Indeed, if the leader is uncharismatic, having factions of principle implies no party unity, whereas party unity is partially achieved with factions of interest. When the leader is charismatic, party unity is always strong with factions of interest, no matter what the ideology of the leader is, whereas with factions of principle the party must be careful to choose a leader with a 'fitting' ideology. Further, the set of ideological positions of the authority charismatic leader which generate no party unity are not the extreme ones, but those around the party’s ideological center.

5 Observable Factional Contributions

The literature on valence politics (see, for example, Schofield 2006) assumes that contributions by factions and activists are visible to voters and taken into account by them. This is also at least partially the case in real world politics. How are our findings modified once we let the factions' contributions be observed by voters and these also care about the mapping from ideological distance between a faction and the leader and contributions? Then we first have to adapt our definition of party unity. Thus, we have strong party unity when the mapping of both contributions and funds or influence shares are strictly decreasing in the distance from the leader. We have weak unity when either the mapping from contributions to distance or that of funds/influence to distance or both are weakly decreasing in distance with the leader, and we have no party unity when the mapping of either contributions or funds/influence shares is non-decreasing in distance with the leader.

Starting with the case of an authority uncharismatic leader, using propositions 1 and 4 above implies immediately that the party fares better with factions of interest, as before. Indeed, with factions of interest, both contributions and fund shares are constant (and
thus weakly decreasing in distance with the leader), whereas with factions of principle both contributions and manifesto influence are strictly increasing in distance between the leader and factions, as \( B \) does not participate in the exchange. We thus have weak party unity with factions of interest, – yielding a party payoff of \( 1/2 \) – but no unity with factions of principle – yielding a party payoff of 0.

What if the leader is authority charismatic? Proposition 2 informs us that, when only two factions of interest contribute, fund shares are strictly decreasing in distance to the leader, but contributions are independent of distance. Thus the party achieves weak party unity and the party payoff is \( \mu(s) + 1/2 \). If all three factions of interest contribute, party unity is always strong, no matter where the leader is located; see Proposition 3: the party payoff is thus maximal and equal to \( \mu(s) + 1 \). Further, as the non-shaded area in the figure below illustrates clearly, for any ideologic positions of the three factions, the party can always find a non-empty set of ideologic positions for the leader such that strong party unity is achieved. Thus allowing voters to observe both contributions and funds shares does not reduce the party’s capacity to achieve party unity when factions are of interest.
Turning to factions of principle, if only two factions contribute under an authority charismatic leader, then party unity is weak because contributions are independent of ideological distance to the leader – see Proposition 4 – and the party payoff is $\mu(s) + 1/2$. Yet, when all three factions contribute, then Proposition 5 informs us that party unity is always either weak or altogether absent, because the mapping from distance to the leader to contributions and/or influence is not monotonically decreasing in distance. The party payoff in this case is either $\mu(s) + 1/2$ or $\mu(s)$.

The figure below summarizes these findings. The light shaded triangular areas are the combinations of positions of $B$ and the leader such that party unity is weak due to either the factions’ contributions or their influence shares, the dark shaded parallelogram represented the set of positions of $B$ and the leader such that unity is absent due to the ordering of either contributions or influence.

All in all, as is clear from the figure, having factions of principle is more problematic for party unity than having factions of interest: with an authority charismatic leader, the party can achieve weak unity at best, but even this comes at the price of selecting a leader that is rather distant from the party’s ideological center and who must often have an ideology that is close to the isolated, minority faction. For example, if $B$ is located at $1$ with $C$, then weak party unity can only be achieved if the leader’s ideology is in $[0, 1/2]$,
and therefore is closer to A than to the other two factions.\footnote{Remark that we are assuming that voters only care about the ranking of factional contributions, not their level. The reasons for this are twofold and mainly pragmatic. First, in our model, there is no obvious way of ranking $V$ and $I$. Thus, we do not know how to compare the level of contributions across factional and leader types. Of course, if we were to take a stand on the ranking of $V$ and $I$, such comparisons could easily be carried out. And, assuming that the values of $V$ and $I$ are not too different, the general picture that would emerge would be that, once again, factions of interest are more conducive to a successful electoral strategy than factions of principle. Second, we believe that the charisma of the party leader and the extent to which a party appears united in front of the electorate are more important determinants of electoral success than the level of total contributions offered by factions. Indeed, a (substantial?) fraction of these contributions are not observable to voters, as they happen behind the party's closed doors. Further, we know of no empirical research on the direct effects of factional contributions on the electoral success of parties. To the contrary, the evidence put forward by, for example, Bittner (2011) suggests that charismatic leaders bring about an unconditional electoral boost to their party and Greene and Haber (2015) find that party disunity reduces the electoral appeal of a party.}

\section{Conclusion}

We developed and analyzed a formal model of the intra-party exchange game the party leader plays with the party factions, to understand the conditions that are most conducive to the party appearing united in front of voters. The leader is either authority charismatic or weak and dull. Factions are either of interest or of principle. Factions are offered party funds or some influence on the party manifesto in exchange for their contributions.

Our results imply that an uncharismatic leader is conducive to weak party unity at
best, irrespective of the type of factions present. Thus having such a leader is a double penalty for the party, as uncharismatic leaders are less valued than charismatic ones, and they do not generate party unity, whereas authority charismatic leaders can. This is especially true when the party’s main factions are of principle, as the dramatic counter-performance of the French socialist party at the last Presidential and legislative elections in France in 2017 shows quite vividly.

We also proved that, when the leader is authority charismatic, party unity is more easily achieved with factions of interest than with factions of principle if voters do not observe the factions’ contributions, and can only be achieved with factions of interest if the electorate observes both the factions’ contributions and their shares of the party funds or manifesto influence. A provocative interpretation of our results is that one of the forces that may push parties to refrain from moving to full, one-man-one-vote internal democracy – and therefore not allow an authority charismatic leader to bias the exchange with factions anymore – is the electoral cost such a move could entail, via the increase in the likelihood that the party appears divided in front of the electorate.¹⁹

If we substantiated the findings of our model with important examples from recent electoral races in several countries such as France, the U.K. and the USA, we believe that our findings could be taken to the data in a more systematic manner, to further our understanding of the mapping from the pre-electoral choices of parties and their leaders to their post-election fates.

Finally, our findings suggest that opening further the blackbox relative to the internal functioning and characteristics of parties, and thus moving firmly away from the assumption of parties as unitary actors, may be a very promising avenue for further research in formal political science, but not only.

¹⁹Of course, internal party democracy has many other important merits.
References


Appendix: Proofs

All the proofs that follow are for $L \in [0, 1/2]$. The proofs for $L \in [1/2, 1]$ are symmetric, one only needs to interchange $C$ and $A$.

Proof of Proposition 1. faction $i = A, B, C$ chooses his contribution to party work by maximizing

$$
\frac{c_i}{\sum_j c_j} V - c_i
$$

The first order condition yields the best response

$$
c_i = \sqrt{(c_j + c_k)V} - c_j - c_k
$$

and thus

$$
c^*_i = \frac{2V}{9} \text{ and } Sh^*_i = \frac{1}{3} \text{ for all } i = A, B, C
$$

Proof of Proposition 2. When only two factions are active, the game reduces to a contest with two players. Faction $i$ chooses his contribution to party work by maximizing

$$
\frac{b_i c_i}{b_i c_i + b_j c_j} V - c_i
$$

The first order condition yields the best response

$$
c_i = \frac{1}{b_i} \sqrt{b_i b_j c_j V} - b_j c_j
$$

and thus

$$
c^*_i = c^*_j = \frac{b_i b_j V}{(b_i + b_j)^2}
$$

Finally, the share of party funds are given by

$$
Sh^*_i = \frac{b_i}{b_i + b_j} \text{ and } Sh^*_j = \frac{b_j}{b_i + b_j}
$$
For \( b_i = 1 - d_{Li} \) and \( b_j = 1 - d_{Lj} \), we have that \( Sh_i^* > Sh_j^* \) if and only if \( d_{Li} < d_{Lj} \). 

\[ \square \]

**Proof of Proposition 3.** Faction \( i = A, B, C \) chooses his contribution to party work by maximizing

\[
\frac{b_i c_i}{\sum_j b_j c_j} V - c_i
\]

The first order condition yields the best response

\[
c_i = \frac{1}{b_i} \left[ \sqrt{b_i (b_j c_j + b_k c_k)} V - b_j c_j - b_k c_k \right]
\]

and thus

\[
c_i^* = \frac{2b_j b_k \left[ b_i (b_j + b_k) - b_j b_k \right] V}{b_i (b_j + b_k) + b_j b_k}
\]

\[
Sh_i^* = \frac{b_i (b_j + b_k) - b_j b_k}{b_i (b_j + b_k) + b_j b_k}
\]

where \( b_i = 1 - d_{Li} \). If \( B < L \), we have the following equilibrium contributions:

\[
c_A^* = \frac{2(1 + B - L) L [(1 - L)^2 + B(1 - 2L)] V}{(1 + B - L^2)^2}
\]

\[
c_B^* = \frac{2 [B + (1 - L)^2] (1 - L)L V}{(1 + B - L^2)^2}
\]

\[
c_C^* = \frac{2(1 + B - L)(1 - L) [(4 - 3L) L - 1 - B(1 - 2L)] V}{(1 + B - L^2)^2}
\]

We have that \( c_A^* > 0 \) and \( c_B^* > 0 \), while \( c_C^* > 0 \) if and only if \( B < \frac{3L^2 - 4L + 1}{2L - 1} \). Observe that the latter condition is never satisfied for \( L \leq 1/3 \), while it is always satisfied for \( L \geq \frac{1}{2}(3 - \sqrt{5}) \).

If \( B > L \), we have the following equilibrium contributions:
\[ c_A^* = \frac{2(1 - B + L)(1 - 3L^2 - B(1 - 2L))V}{[B - 1 - (2 - L)L]^2} \]

\[ c_B^* = \frac{2(1 - L)L(1 - B + L^2)V}{[B - 1 - (2 - L)L]^2} \]

\[ c_C^* = \frac{2(1 - B + L)(L - 1)(1 - L(2 + L) - B(1 - 2L))V}{[B - 1 - (2 - L)L]^2} \]

We have that \( c_A^* > 0 \) and \( c_B^* > 0 \), while \( c_C^* > 0 \) if and only if \( B > \frac{L^2 + 2L - 1}{2L - 1} \). Observe that the latter condition is always satisfied for \( L \geq \frac{1}{2}(3 - \sqrt{5}) \).

Consider the parameter configurations in which all three factions are contributing to party work:

1. \( L \leq \frac{1}{3} \) and \( B > \frac{L^2 + 2L - 1}{2L - 1} \)

2. \( \frac{1}{3} < L < \frac{1}{2}(3 - \sqrt{5}) \) and \( \frac{L^2 + 2L - 1}{2L - 1} < B < \frac{3L^2 - 4L + 1}{2L - 1} \)

3. \( \frac{1}{2}(3 - \sqrt{5}) < L \leq \frac{1}{2} \)

If \( B < L \), we have

\[ c_A^* - c_B^* = \frac{2BL[1 + B(1 - 2L) + L(3L - 4)]V}{(1 + B - L^2)^2} < 0 \]

\[ c_A^* - c_C^* = \frac{2[B + (1 - L)^2](1 + B - L)(1 - 2L)V}{(1 + B - L^2)^2} > 0 \]

\[ c_B^* - c_C^* = \frac{2(1 + B - 2L)(1 - L)[(1 - L)^2 + B(1 - 2L)]V}{(1 + B - L^2)^2} > 0 \]

If \( B > L \), we have

\[ c_A^* - c_B^* = \frac{2(2L - B)L(1 - L(2 + L) - B(1 - 2L))V}{[B - 1 - (2 - L)L]^2} > 0 \] if and only if \( B > 2L \)

\[ c_A^* - c_C^* = \frac{2(1 - B + L)(1 - 2L)(1 - B + L^2)}{[B - 1 - (2 - L)L]^2} > 0 \]
\[ c_B^* - c_C^* = \frac{2(1 - B)(1 - L)[1 - B(1 - 2L) - 3L^2]V}{[B - 1 - (2 - L)L]^2} > 0 \]

and thus:

- \( c_A^* > c_B^* > c_C^* \) if \( B > 2L \)
- \( c_B^* > c_A^* > c_C^* \) if \( B < 2L \)

We know that for faction \( i = A, B, C \), the equilibrium share of party funds is given by

\[ Sh_i^* = \frac{b_i(b_j + b_k) - b_j b_k}{b_i(b_j + b_k) + b_j b_k} \]

Hence, it follows directly that \( Sh_i^* > Sh_j^* \) if and only if \( b_i > b_j \). Therefore, as the biases are linear in the distance:

- \( Sh_A^* > Sh_B^* > Sh_C^* \) if \( B > 2L \)
- \( Sh_B^* > Sh_A^* > Sh_C^* \) if \( B < 2L \)

\[ \square \]

**Proof of Proposition 4.** Faction \( i = A, B, C \) chooses his contribute by maximizing

\[ \sum_m c_i c_m I + \frac{c_j}{\sum_m c_m} I(1 - d_{ij}) + \frac{c_k}{\sum_m c_m} I(1 - d_{ik}) - c_k \]

The first order condition yields the best response

\[ c_i = \sqrt{(d_{ij}c_j + d_{ik}c_k)I - c_j - c_k} \]

and thus

\[ c_i^* = \frac{2d_{ij}d_{ik}(d_{ij} + d_{ik} - d_{jk})d_{jk}^2 I}{[d_{ij}^2 + (d_{ik} - d_{jk})^2 - 2d_{ij}(d_{ik} + d_{jk})]^2} \]

Given that \( d_{AB} = B, d_{BC} = 1 - B \) and \( d_{AC} = 1 \), this yields:

\[ c_A^* = c_C^* = \frac{I}{4} \] and \( c_B^* = 0 \)
and the equilibrium individual influences on the party manifesto are given by

\[ Sh_A^* = Sh_C^* = \frac{1}{2} \text{ and } Sh_B^* = 0 \]

**Proof of Proposition 5.** Whenever one faction does not contribute to party work, the game reduces to a contest with two players. Faction \( i = A, B, C \) chooses his contribution by maximizing

\[
\frac{b_i c_i}{b_i c_i + b_j c_j} I + \frac{b_j c_j}{b_i c_i + b_j c_j} I(1 - d_{ij}) - c_i
\]

The first order condition yields the best response

\[ c_i = \frac{1}{b_i} \sqrt{b_i b_j d_{ij} I} - b_j c_j \]

and thus

\[ c_i^* = c_j^* = \frac{b_i b_j d_{ij} I}{(b_i + b_j)^2} \]

Finally, the individual influences on the party manifesto are given by

\[ Sh_i^* = \frac{b_i}{b_i + b_j} \text{ and } Sh_j^* = \frac{b_j}{b_i + b_j} \]

As \( b_i = 1 - d_{Li} \) and \( b_j = 1 - d_{Lj} \), we have that \( Sh_i^* > Sh_j^* \) if and only if \( d_{Li} < d_{Lj} \). 

**Proof of Proposition 6.** Faction \( i = A, B, C \) chooses his contribution by maximizing

\[
\frac{b_i c_i}{\sum_m b_m c_m} I + \frac{b_j c_j}{\sum_m b_m c_m} I(1 - d_{ij}) + \frac{b_k w_k}{\sum_m b_m c_m} I(1 - d_{ik}) - c_i
\]

The first order condition yields the best response

\[ c_i = \frac{1}{b_i} \left[ \sqrt{b_i (d_{ij} b_j c_j + d_{ik} b_k c_k) I} - b_j c_j - b_k c_k \right] \]

and thus
\[ c_i^* = \frac{2b_ib_kd_{ij}d_{ik}d_{jk}^2(b_jb_jd_{ij} + b_ib_kd_{ik} - b_jb_kd_{jk})I(L)}{[b_i(d_{ij} - d_{ik})(b_jd_{ij} - b_kd_{ik}) - (b_j(b_i + b_k)d_{ij} + b_k(b_i + b_j)d_{ik})d_{jk} + b_jb_kd_{jk}^2]^2} \]

\[ Sh_i^* = \frac{d_{jk}[b_ib_kd_{jk} - b_i(b_jd_{ij} + b_kd_{ik})]}{b_i(d_{ij} - d_{ik})(b_jd_{ij} - b_kd_{ik}) - (b_j(b_i + b_k)d_{ij} + b_k(b_i + b_j)d_{ik})d_{jk} + b_jb_kd_{jk}^2} \]

If \( B < L \), we have the following equilibrium contributions:

\[ c_A^* = \frac{(1 + B - 2L)L}{2(1 + B - L)} \]

\[ c_B^* = \frac{(1 - L) L [1 + B(1 - 2L) - 2(1 - L)L]}{2(1 - B)(1 + B - L)^2} \]

\[ c_C^* = \frac{(1 - L) [2(1 - L)L - B(1 + B - 2L)]}{2(1 - B)(1 + B - 2L)} \]

and thus \( c_i^* > 0 \) for all \( i = A, B, C \).

If \( B > L \), we have the following equilibrium contributions:

\[ c_A^* = \frac{L [2L(B - L) - (1 - B)B]}{2B(1 - B + L)} \]

\[ c_B^* = \frac{(1 - L) [B - 2L(B - L)]}{2B(1 - B + L)^2} \]

\[ c_C^* = \frac{(2L - B)(1 - L)}{2(1 - B + L)} \]

and thus \( c_A^* > 0 \) and \( c_B^* > 0 \), while \( c_C^* > 0 \) if and only if \( B < 2L \).

Consider the parameter configurations such that all three factions are active, hence \( B < 2L \). If \( B < L \) we have:
\[ c_A^* - c_B^* = \frac{(B - L)[2L(1 + B - L) - B(1 + B)]I}{2(1 - B)(1 + B - L)^2} < 0 \]

\[ c_A^* - c_C^* = \frac{(L - B)[2L(1 + B - L) - (1 + B)]I}{2(1 - B)(1 + B - L)} < 0 \]

\[ c_B^* - c_C^* = \frac{(1 + B)(1 + B - 2L)(L - B)(1 - L)I}{2(B - 1)(1 + B - L)^2} < 0 \]

If \( B > L \) we have:

\[ c_A^* - c_B^* = \frac{(2 - B)(2L - B)(B - L)L_I}{2B(1 - B + L)^2} > 0 \]

\[ c_A^* - c_C^* = \frac{(B - L)[B(1 - 2L) + 2L^2]I}{2B(1 + L - B)} > 0 \]

\[ c_B^* - c_C^* = \frac{(B - L)(1 - L)[B(1 - B) + 2L(B - L)]I}{2B(1 - B + L)^2} > 0 \]

and thus:

1. If \( B < L \), we have \( c_C^* > c_B^* > c_A^* \)

2. If \( B > L \), we have \( c_A^* > c_B^* > c_C^* \)

Further, if \( B < L \), the equilibrium influences on the party manifesto are given by

\[ Sh_A^* = \frac{1 + B - 2L}{2(1 + B - L)} \]

\[ Sh_B^* = \frac{(B + 1) - 2L(1 + B - L)}{2(1 - B)(1 + B - L)} \]

\[ Sh_C^* = \frac{2L(1 + B - L) - B(1 + B)}{2(1 - B)(1 + B - L)} \]

and thus we have

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\[ Sh_A^* - Sh_B^* = \frac{2L(2B - L) - B(B + 1)}{2(1 - B)(1 + B - L)} < 0 \]

\[ Sh_A^* - Sh_C^* = \frac{2L(L-2)+(1+B)}{2(1-B)(1-L)-L} > 0 \text{ if and only if } B > 4L - 2L^2 - 1 \]

\[ Sh_B^* - Sh_C^* = \frac{(1+B-2L)^2}{2(1-B)(1+B-L)} > 0 \]

Finally, if \( B > L \), the equilibrium influences on the party manifesto are given by

\[ Sh_A^* = \frac{2L(B-L) + B(1-B)}{2B(1-B+L)} \]

\[ Sh_B^* = \frac{B - 2L(B-L)}{2B(1-B+L)} \]

\[ Sh_C^* = \frac{2L - B}{2(1-B+L)} \]

and thus we have

\[ Sh_A^* - Sh_B^* = -\frac{(B - 2L)^2}{2B(1-B+L)} < 0 \]

\[ Sh_A^* - Sh_C^* = \frac{B - 2L^2}{2B(1-B+L)} > 0 \]

\[ Sh_B^* - Sh_C^* = \frac{B(1+B) - 2L(2B - L)}{2B(1-B+L)} > 0 \]

Therefore:

- If \( B < 4L - 2L^2 - 1 \), we have \( Sh_B^* > Sh_C^* > Sh_A^* \)
- If \( B > 4L - 2L^2 - 1 \), we have \( Sh_B^* > Sh_A^* > Sh_C^* \)