Supplementary material: Participation quorums in costly meetings

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1 Appendix

Proof of Proposition 1. Consider any zero-quorum meeting.[1] We show that if in equilibrium $|A^*| \neq 0$ then: (1) the number of attendees is even, (2) there are no gaps between any two consecutive attendees at each side of 1/2, (3) the individuals located at 0 and 1 attend, (4) there is an equal number of attendees at each side of 1/2, and (5) such equilibrium is characterized by unique attendance thresholds.

1) *[Even Number of Attendees]* We proceed by contradiction. Suppose the number of attendees is odd and if individual m exits then $x_m(A') = x_m(A)$. Then, m exits if and only if $V(0) > V(0) - c$ or, equivalently, $c > 0$. Therefore, m always exits.

Suppose the number of attendees is odd (as in Figure 1) and if individual m exits then $x_m(A') \neq x_m(A)$. If c is such that m has no incentive to exit, consider non-attendee i whose location is such that by attending, the policy moves to the same location as it does if m exits. Let the distance between individual m and the new policy $x_m(A')$ in case he exits be x. It follows that the distance between the non-attendee i and the policy $x_m(A)$ is $y = 2x$. If m attends, it means that $-V(x) \leqslant -V(0) - c$, or, equivalently, $c \leqslant V(x)$. Now, notice that i is willing to attend if and only

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[1] The results in Proposition 1 refer to meetings $j = 0, 2$. To avoid heavy notation we omit subscript j in points 1 to 4 since we refer to any of the two meetings. In point 5 we introduce such subscript when we refer to one of the two meeting in particular.

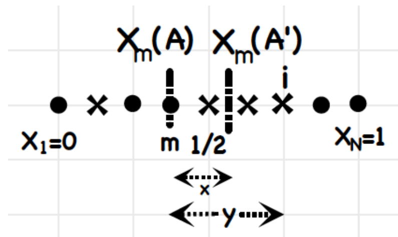

Figure 1: Proof of point 1.

if $-V(x) - c \geqslant -V(y)$, or, equivalently, $c \leqslant V(y) - V(x)$. As $y = 2x$ and $V(.)$ is strictly convex, it follows that $V(x) < -V(x) + V(y)$, or, equivalently, $V(2x) > 2V(x)$. Therefore, i attends. Therefore, the equilibrium number of attendees cannot be odd.

2) *[No Gaps]* Given the exit process, we know that at any stage, the first attendee who exits (if any) is the one with the lowest potential disutility from doing so. Beginning with the full set of attendees N, where N is odd, we know that for $c > 0$, the first attendee to exit is m. Let l and r be the first attendees on the left and on the right of $x_m(N)$ respectively.[2] The potential disutility from exiting is the same for both of them, and is given by $V(2d) - V(d)$. Consider now the next attendees on the left of l and on the right of r respectively. Their disutility from exiting is identical, and is given by $V(3d) - V(2d)$, which is strictly higher than $V(2d) - V(d)$ given the strict convexity of $V(.)$. Clearly, this is also true for any pair of attendees who are located even further from $x_m(N)$. Therefore, the first one to exit is either l or r. From there on, if c is high enough for the exit process to keep going:

a) At any stage in which the number of attendees is odd, either m, or one (and only one) of the attendees l or r is the first one to exit. Given the exit process, a situation in which the number of attendees is odd is such that m is next to r (l), while, as some attendees have already exited, there are gaps between m and l (r). Suppose, w.l.o.g., that m is next to r, and let kd be the distance between m and l. We aim at showing that the first individual to exit is either m or l (that is, either the median attendee or the furthest one next to him). If m exits, his disutility from doing so is given by $V(\frac{(k-1)d}{2})$. If r exits, he will suffer a disutility of $V(\frac{kd}{2} + d) - V(d)$. As $V(\frac{(k-1)d}{2}) = V(\frac{(k+1)d}{2} - d) < V(\frac{(k+1)d}{2}) - V(d)$, it follows that m always exits before r. Clearly,

[2] Observe that assuming that N is odd is without loss of generality, since assuming instead that N is even simply means that we start from here on.

this is also true for any attendee on the right of r. Indeed, if any such attendee exits, the effect on the policy will be exactly the same as if r exits, while, being strictly further, the disutility from doing so in terms of distance is strictly higher than the one of r by the strict convexity of $V(.)$. Now, if l exits, his disutility from doing so is given by $V((k+\frac{1}{2})d) - V(kd)$. We do not know whether it is m or l who has the lowest disutility from exiting. However, by the same reasoning as the one just described above, we know that any attendee on the left of l suffers a strictly higher disutility from exiting than l does, so that l always exits first. Therefore, either m or l exits first.

b) At any stage in which the number of attendees is even, so that the policy is $x_m(A) = \frac{(x_l+x_r)}{2}$, either l or r is the first one to exit. Let kd be the distance between i and $x_m(A)$, $i = l, r$. The disutility from exiting for individual $i = l, r$ is given by $V(2kd) - V(kd)$. Consider now the next attendees on the left of l and on the right of r respectively. Their disutility from exiting is strictly higher than $V(2kd) - V(kd)$ given the strict convexity of $V(.)$. Furthermore, this is also true for any pair of attendees who are located even further from $x_m(A)$.

3) *[Extremists at 0 and 1 attend]* Consider any stage of the exit process such that there are still attendees on both sides of 1/2. As just shown, regardless of whether the number of attendees is even or odd, the first one to exit is always either m, or l, or r. Therefore, at any such stage (i.e., such that there are attendees on both sides of 1/2), the attendee located at 0 (respectively 1) has a strictly higher disutility from exiting than l (respectively r). Hence, at any such stage, both the individuals located at 0 and 1 attend.

4) *[Balanced]*

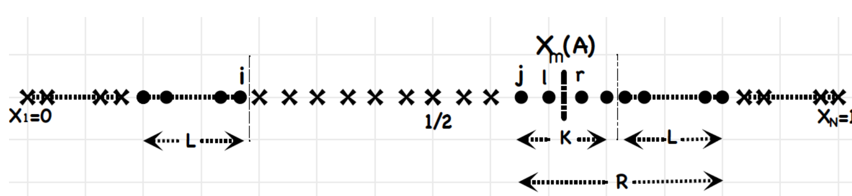

Figure 2: Proof of point 4.

Let $L \geqslant 0$ be the number of attendees on the left of 1/2 (see Figure 2 for illustration). Similarly, let R be the number of attendees on the right of 1/2, and assume w.l.o.g. that $R > L$, and so $R = L + K$, where $K > 0$ and is even by point 1. From point 2, it follows that the L and R

attendees respectively on the left and on the right of 1/2 are consecutive (i.e., there are no gaps between them). Let $x_m(A)$ be the chosen policy given this set of attendees, which is located at the median of the K attendees. Observe that if l and r attend, it means that c is small enough so that it is worth attending to prevent $x_m(A)$ from moving by $\frac{1}{2}d$, that is, it means that $c \leqslant V(d) - V(\frac{1}{2}d)$. Let i and j be the first attendees on the left and on the right of 1/2 respectively, and consider any non-attendee between i and j. If any such non-attendee were to attend, the policy would move to x_l, and he would do so if and only if $c \leqslant V((k+\frac{1}{2})d) - V(kd)$, where $kd > d$ is the distance between any non-attendee and l. It turns out that if r attends, any abstainer between i and j wants to attend as well, since $V((k+\frac{1}{2})d) - V(kd) > V(d) - V(\frac{1}{2}d)$ for all $k > 1$ by the strict convexity of $V(.)$. Therefore, the equilibrium is such that $K = 0$ (i.e., balanced).

5) *[Unique Thresholds]* Let t_0 be the unique solution of $c = V(2t_0) - V(t_0)$, and t_2 the unique solution of $c = \beta[V(2t_2) - V(t_2)]$. The threshold t_j, $j = 0, 2$, defines a set of attendees $A_j^* = \{x_i : x_i \in [0, \frac{1}{2} - t_j] \cup [\frac{1}{2} + t_j, 1]\}$ that satisfies points 1 to 4 (see Figure 1 in the main body of the paper). Such a situation is an equilibrium since none of the attendees has an incentive to exit and none of the abstainers has an incentive to attend. Notice that the attendees with the highest potential benefit from exiting are the ones with ideal points x_l and x_r. By the definition of t_j, $b_l(A_j^*) = b_r(A_j^*) < 0$, and hence both individuals located at x_l and x_r attend. Furthermore, by the definition of t_j it holds that $b_i(A_j^*) > 0$ for any i with $x_i \in (\frac{1}{2} - t_j, \frac{1}{2} + t_j)$, and hence none of the abstainers is willing to attend. Finally, given that the threshold t_j is uniquely defined, the equilibrium is unique.

In order to derive the equilibrium number of attendees $|A_j^*|$, observe that if $t_j > 1/2$ then $|A_j^*| = 0$ as the attendees located at x_1 and x_N have no incentive to attend. If $t_j \leq 1/2$, the exit process stops at the individual located at $k_j d$ (i.e., attendee x_l), who is the first attendee (on the left of 1/2) for whom $b_i(A_j^*) \leqslant 0$. Formally, k_j is the maximum natural number such that $k_j \leqslant (\frac{1}{2} - t_j)(N-1)$. Therefore, the equilibrium number of attendees on the left of 1/2 is given by $k_j + 1$, and thus $|A_j^*| = 2(k_j + 1)$. Given the symmetric distribution of $x_i \in A_j^*$, the equilibrium policy is $x_m(A_j^*) = 1/2$.

□

Proof of Proposition 2. As t_0 is the unique solution of $c = V(2t_0) - V(t_0)$, and t_2 is the unique solution of $c = \beta\,[V(2t_2) - V(t_2)]$, we have that

$$\beta = \frac{V(2t_0) - V(t_0)}{V(2t_2) - V(t_2)}$$

As $\beta \leqslant 1$, it follows that $t_2 \geqslant t_0$, and thus $k_2 \leqslant k_0$ and $|A_2^*| \leqslant |A_0^*|$. □

Proof of Proposition 3. a) Suppose that $Q < |A_0^*|$, and consider attendees x_l and x_r of the zero-quorum game for given N and c. We know that $b_l(A_0^*) = b_r(A_0^*) \leqslant 0$, the set E is empty, and there are $|A_0^*|$ attendees in equilibrium. Let us now introduce a quorum $Q < |A_0^*|$. All moderates located between x_l and x_r of the zero-quorum game exit, as before. Then, $b_i(A_1^*) \leqslant 0$ for all $i \in A_1^*$, the set E is empty, so that the exit process stops at x_l and x_r. Therefore, $|A_1^*| = |A_0^*|$, and $x_m(A_1^*) = x_m(A_0^*) = 1/2$. That is, the attendance decision of any individual is the same as in the zero-quorum case —in particular, none of the $|A_1^*|$ individuals can affect whether the quorum requirement is met— and hence the equilibrium is unaffected.

b) Suppose that $Q = |A_0^*|$. In this case, the quorum is even, and all individuals between $x_l = (\frac{Q}{2} - 1)d$ and $x_r = 1 - (\frac{Q}{2} - 1)d$ exit, since the presence of a quorum requirement does not alter their cost-benefit calculation during the exit process.[3] Once all moderates in (x_l, x_r) have exited, two blocks of $Q/2$ consecutive individuals in $[0, x_l]$ and $Q/2$ consecutive individuals in $[x_r, 1]$ are formed. To fix ideas, we are at a situation where a set of Q individuals has not exited yet, and the exit process may go on further. Formally, we denote the set of the remaining Q attendees as $\mathbf{Q} = \{x_i : x_i \in [0, x_l] \cup [x_r, 1]\}$. From here on, the cost-benefit calculation of these Q individuals alters compared to the zero-quorum game.

Any individual, by exiting, prevents the quorum to be met and hence postpones the decision. If the meeting is postponed, we know from Proposition 1 that the policy outcome is $x_m(A_2^*) = 1/2$, and all individuals belonging to the set $A_2^* = \{x_i : x_i \in \left[0, \frac{1}{2} - t_2\right] \cup \left[\frac{1}{2} + t_2, 1\right]\}$ attend the second meeting. Notice that since $|A_2^*| \leqslant |A_0^*|$, $Q = |A_0^*|$, and given the structure of the equilibrium (Proposition 1), it holds that $A_2^* \subseteq \mathbf{Q}$.

[3] See the first graph in Figure 2 in the main body of the paper.

Clearly, any individual $i \in A_2^*$ has no incentive to exit, since by doing so, he postpones the (same) decision while still paying the attendance cost. Formally, for any $i \in A_2^*$, it holds that

$$b_i(\mathbf{Q}) = U_i^{Exit} - U_i^{Attend} = \beta \left[1 - V(|x_i - \frac{1}{2}|)\right] - c - \left[1 - V(|x_i - \frac{1}{2}|) - c\right] \leqslant 0$$

Conversely, any individual belonging to $\mathbf{Q}$, but not belonging to A_2^*, may be willing to exit in order to save the cost of attending the second meeting, where he knows that the same decision will be made.

First, we look for the condition such that the set of potential free-riders $\mathbf{Q} \setminus A_2^*$ is non-empty. This holds if at least individual l (or r) does not have an incentive to attend the second meeting. That is, $\mathbf{Q} \setminus A_2^*$ is non-empty if and only if

$$\beta < \frac{c}{V(|1-2kd|) - V(|kd - \frac{1}{2}|)} = \beta_2$$

If $\mathbf{Q} \setminus A_2^*$ is empty (i.e., $\beta \geqslant \beta_2$), then the decision is made in the first meeting, so that $|A_1^*| = Q$ and $x_m(A_1^*) = x_m(A_0^*) = 1/2$.

If $\mathbf{Q} \setminus A_2^*$ is non-empty (i.e., $\beta < \beta_2$), we have to look for the conditions such that the exit process goes further. Formally, for any individual $i \in \mathbf{Q} \setminus A_2^*$, it holds that

$$b_i(\mathbf{Q}) = U_i^{Exit} - U_i^{Attend} = \beta \left[1 - V(|x_i - \frac{1}{2}|)\right] - \left[1 - V(|x_i - \frac{1}{2}|) - c\right]$$
$$= (\beta - 1)\left[1 - V(|x_i - \frac{1}{2}|)\right] + c$$

Given that $V(.)$ is strictly increasing, if $b_i(\mathbf{Q}) > 0$ for some i, it follows that the individuals with the highest (equal) incentives to exit (i.e., i with the highest value $b_i(\mathbf{Q})$) are the ones located at x_e and $1 - x_e$. Hence, the exit process continues further than $\mathbf{Q}$ if and only if the latter individuals are willing to exit, which is true if and only if

$$\beta > 1 - \frac{c}{1 - V(|x_e - \frac{1}{2}|)} = \beta_1$$

If $\beta > \beta_1$, after individual x_e exits, all remaining individuals exit as well since the quorum cannot be met, so that $|A_1^*| = 0$, and from Proposition 1, $|A_2^*| = 2(k_2 + 1)$ and $x_m(A_2^*) = 1/2$.

If $\beta \leqslant \beta_1$, then the potential free-rider with the highest incentive to exit (i.e., w.l.o.g. the one located at x_e) prefers to attend the first meeting to fulfill the quorum, so that $|A_1^*| = Q$ and $x_m(A_1^*) = x_m(A_0^*) = 1/2$.

c) Suppose that $Q > |A_0^*|$ is **even**. Similar to the case of $Q = |A_0^*|$, all moderate individuals between $x_l = (\frac{Q}{2} - 1)d$ and $x_r = 1 - (\frac{Q}{2} - 1)d$ exit, since the presence of a quorum requirement does not alter their cost-benefit calculation during the exit process.[4] To fix ideas, we are at a situation where a set of Q individuals has not exited yet, and the exit process may go on further.

Contrary to the case of $Q = |A_0^*|$, since $Q > |A_0^*|$ and $|A_0^*| \geqslant |A_2^*|$, it holds that $Q > |A_2^*|$, which means that the set of free-riders $\mathbf{Q} \setminus A_2^*$ is non-empty. Now, any individual belonging to $\mathbf{Q}$, but not belonging to A_2^*, may be willing to exit in order to save the cost of attending the second meeting, where he knows that the same decision will be made. In the same way as for $Q = |A_0^*|$, we conclude that if $\beta > \beta_1$, the individual located at x_e exits, all remaining individuals exit as well since the quorum cannot be met, so that $|A_1^*| = 0$, $|A_2^*| = 2(k_2 + 1)$ and $x_m(A_2^*) = 1/2$.

Conversely, if $\beta \leqslant \beta_1$, then $b_i(\mathbf{Q}) \leqslant 0$ for all $i \in \mathbf{Q}$, the exit process stops, none of the non-attendees has an incentive to attend (they have just exited), so that $|A_1^*| = Q$ and $x_m(\mathbf{Q}) = 1/2$.

d) Suppose that $Q > |A_0^*|$ is **odd**. All moderate individuals between $x_l = (\frac{Q-1}{2})d$ and $x_r = 1 + \left[1 - (\frac{Q-1}{2})\right] d$ exit as in the zero-quorum game, since the presence of a quorum requirement does not alter their cost-benefit calculation during the exit process.[5] Once all moderates in (x_l, x_r) have exited, two blocks of $(Q + 1)/2$ consecutive individuals in $[0, x_l]$, and $(Q - 1)/2$ consecutive individuals in $[x_r, 1]$ are formed. To fix ideas, we are at a situation where a set of Q individuals has not exited yet, and the exit process may go on further.

Any individual, by exiting, prevents the quorum to be met and hence postpones the decision. If the meeting is delayed, we know from Proposition 1 that the policy outcome is $x_m(A_2^*) = 1/2$, and all individuals belonging to the set $A_2^* = \{x_i : x_i \in \left[0, \frac{1}{2} - t_2\right] \cup \left[\frac{1}{2} + t_2, 1\right]\}$ attend the second meeting. Notice that when the set of attendees is $\mathbf{Q}$, then $x_m(\mathbf{Q}) = (\frac{Q-1}{2})d$, and hence the policy is distorted. This, in turn, implies that there are incentives to exit either to return the policy to $1/2$ in the next

[4]See the second graph in Figure 2 in the main body of the paper.
[5]See the third graph in Figure 2 in the main body of the paper.

meeting (since $x_m(A_2^*) = 1/2$), or —as in the case of an even quorum— to save the cost of attending the second meeting.

Let us consider first the individual with the highest incentive to exit in order to save the attendance cost, that is, we focus on the set of potential free-riders $\mathbf{Q} \setminus A_2^*$. Notice that since $Q > |A_0^*| \geqslant |A_2^*|$, the latter set is non-empty. Any of these individuals, by exiting, postpones the decision (which is harmful), but saves the cost of attending the second meeting, where he knows that the policy will be 1/2. Formally, for any $i \in \mathbf{Q} \setminus A_2^*$, it holds that

$$b_i(\mathbf{Q}) = U_i^{Exit} - U_i^{Attend} = \beta \left[1 - V(|x_i - \frac{1}{2}|)\right] - \left[1 - V(|x_i - (\frac{Q-1}{2})d|) - c\right]$$

If $b_i(\mathbf{Q}) \leqslant 0$ for all $i \in \mathbf{Q} \setminus A_2^*$, then no incentives to exit so as to save the attendance cost are present. Similar to the even case, we have that $b_i(\mathbf{Q}) > 0$ if and only if

$$\beta > \frac{[1 - V(|x_i - (\frac{Q-1}{2})d|) - c}{[1 - V(|x_i - \frac{1}{2}|)]}$$

If the latter condition holds for some i, the value of $b_i(\mathbf{Q})$ is maximized for the individual located at x_e given that $V(.)$ is strictly increasing. Hence, in order for the exit process to go further than $\mathbf{Q}$, with individual x_e exiting, it must hold that

$$\beta > \frac{[1 - V(|x_e - (\frac{Q-1}{2})d|) - c}{[1 - V(|x_e - \frac{1}{2}|)]} = \beta_3$$

Let us now consider the individual with the highest incentive to exit so as to return the policy to 1/2 (recall that at this stage, the policy is distorted since $x_m(\mathbf{Q}) = (\frac{Q-1}{2})d < \frac{1}{2} = x_m(A_2^*)$). The distortion being on the left, the individual with the highest incentive to exit is the one located at $x_N = 1$, since he's the furthest away from the policy at that stage. Formally, for any $i \in \mathbf{Q} \cap A_2^*$,

$$b_i(\mathbf{Q}) = U_i^{Exit} - U_i^{Attend} = \beta \left[1 - V(|x_i - \frac{1}{2}|)\right] - \left[1 - V(|x_i - (\frac{Q-1}{2})d|)\right]$$

If $b_i(\mathbf{Q}) \leqslant 0$ for all $i \in \mathbf{Q} \cap A_2^*$, then no individual exits so as to return the policy to 1/2. Conversely, if $b_i(\mathbf{Q}) > 0$ for some $i \in \mathbf{Q} \cap A_2^*$, then $b_i(\mathbf{Q})$ is maximized for the individual located at $x_N = 1$.

Formally, the latter exits if and only if

$$\beta > \frac{1 - V(1 - (\frac{Q-1}{2})d)}{1 - V(1 - \frac{1}{2})} = \beta_4$$

Suppose that $\beta \leqslant \min\{\beta_3, \beta_4\}$, so that both the individuals located at x_e and 1 attend. In that case, the exit process stops here, and since no individual in (x_l, x_r) has an incentive to attend, we are at an equilibrium. The equilibrium number of attendees is given by $|A_1^*| = Q$, and the equilibrium policy is $x_m(\mathbf{Q}) = (\frac{Q-1}{2})d$. If $\beta > \min\{\beta_3, \beta_4\}$, then either the individual located at x_e or the one located at $x_N = 1$ exits (or both if they coincide, that is, if the individual located at $x_N = 1$ is a free-rider), all remaining attendees exit as well, so that $|A_1^*| = 0$. From Proposition 1, $|A_2^*| = 2(k_2 + 1)$ and $x_m(A_2^*) = 1/2$.

$\square$