Sequential Choice of Sharing Rules in Collective Contests*

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Abstract

Groups competing for a prize need to determine how to distribute it among their members in case of victory. We show that the timing of such groups' internal organization has important implications that depend on the nature of the prize. When the prize is sufficiently private both groups actively take part in the competition and switching from a simultaneous to a sequential timing where the small group is the leader consists in a Pareto improvement and reduces aggregate effort expenditures. On the contrary, when the large group is the leader aggregate effort increases. These differences stem from the fact that while the sharing rules are strategic complements from the perspective of the large group, they are strategic substitutes from the perspective of the small one. When the prize is not private enough, only the large group is active in the competition and switching from a simultaneous to a sequential timing may reverse the results in terms of aggregate effort. Interestingly, the sequentiality of moves eliminates the group size paradox regardless of the leader's size, hence the small group never outperforms the large one even when it has the leadership advantage.

Keywords: collective rent seeking, sequential, group size paradox, sharing rules, strategic complements, strategic substitutes

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1 Introduction

Contests among organizations and groups of individuals are widespread. Examples include research and development races, pre-electoral campaigns, procurement contests, or even sport and art contests.[1] In all these contests, groups' performance depends on individual contributions of their members, which implies that groups need to coordinate and establish some rules regarding their internal organization. As the literature on collective contests has pointed out, a key element of groups' organization is related to the allocation of the prize among the winning group members.[2]

While contests among groups are generally thought as simultaneous, the timing in which contenders organize and implement their internal rules need not necessarily be so. Indeed, there is no *a priori* reason to believe that before the actual competition takes place, the timing in which the involved organizations decide upon their governing rules coincides. Differences in the size or in the informational advantage of organizations, as well as the existence of some established organizations challenged by an entrant, may result in the sequential determination of their governing rules. Consequently, groups involved in a simultaneous competition may well be deciding upon their internal rules in a sequential fashion. Similar to previous literature on contests among individuals, we find that if the order of moves is determined endogenously, the equilibrium timing of the game is a sequential one, and this very fact constitutes a strong justification for departing from the simultaneity assumption (Baik and Shogren, 1992; Leininger, 1993; Morgan, 2003).

In this paper, we contribute to the literature on collective rent seeking by analyzing how the order of moves regarding the choice of group sharing rules may alter the structure of competition and individual incentives to provide effort. We analyze a situation in which two groups compete for a mixed public-private prize, while the sharing rules determining

[1]For surveys of the literature on contests in general, see among others Corchón (2007), Konrad (2009), and also Dechenaux et al. (2015) for the related experimental evidence. On collective contests, see the recent survey by Kolmar (2013), and also Sheremeta (2015) for a recent review of experimental evidence.

[2]Starting with Nitzan (1991), the literature has considered both exogenous and endogenous sharing rules, while it has assumed that the choice of such rules may occur under either public or private information. For a recent survey on prize-sharing rules in collective rent seeking, see Flamand and Troumpounis (2015).

the allocation of the private part of the prize within the group may exhibit varying degrees of meritocracy, potentially allowing for transfers among the groups' members. As it turns out, depending on the exact nature of the contested prize, the timing of the interaction may be key to the dynamics of individual incentives and thus to group performance. The main driving force of our results is the fact that while the degrees of meritocracy of the sharing rules are strategic substitutes from the perspective of the small group, they are strategic complements from the perspective of the large group —hence the timing of actions matters.

We show that the sequential choice of sharing rules fully eliminates the group size paradox (GSP henceforth), a situation in which a smaller group outperforms a larger one in terms of winning probabilities (Olson, 1965). In contrast, when the group sharing rules are chosen simultaneously, and provided that they allow for transfers among group members, the GSP arises whenever the prize is sufficiently private (Balart et al., 2016). With sequential sharing rules, in fact, the two groups achieve the same probability of winning the prize whenever the latter is either a purely public or a purely private good. For all intermediate levels of privateness, the large group outperforms the small one. If the large group is the leader, the intuition is similar to the standard Stackelberg duopoly model. That is, the large group commits to a more meritocratic sharing rule than in the simultaneous game (hence effort increases), while the small group selects a less meritocratic rule than in the simultaneous game (hence effort decreases), which prevents the occurrence of the GSP. The small group acting as the leader follows a very different strategy. It commits to a less meritocratic rule than in the simultaneous game, thereby weakening competition between groups. In turn, the large group also responds with a less meritocratic rule than in the simultaneous game, but the reduction is relatively lower than the one implemented by the small group. As a result, total effort decreases relatively less in the large group, which again eliminates the GSP.

As when the group sharing rules are chosen simultaneously, the exact nature of the prize is key to group's performance. Indeed, we show that when the prize is not private enough and the sharing rules allow for transfers, allowing either group to act as the leader leads to monopolization, as in the simultaneous case. In the context of two-groups contests, monopolization refers to a situation in which one group retires from the competition (Ueda, 2002). Interestingly, the large group acting as the first mover takes advantage of its leadership by preventing the small group from being active for a greater range of privateness of the prize compared to the simultaneous case, thereby making monopolization more likely. In contrast, the small group is unable to take advantage of the timing structure when it is the leader, so monopolization occurs in the same instances as in the simultaneous game. In contrast to what happens when both groups are active, when the small group is inactive switching from a simultaneous to a sequential timing reduces aggregate effort. Thus, the degree of privateness of the prize is a critical feature that should be taken into account by the contest designer.

Litigation is one example that our model can be applied to. Since parties involved in a legal battle spend irretrievable resources to prevail in court, litigation has often been modeled as a rent-seeking contest (Baye et al., 2005; Farmer and Pecorino, 1999; Gürtler and Kräkel, 2010). Many law firms make use of incentive pay, conditioning lawyers' compensation on their individual performance (for example, through bonus payments), which is typically measured by the number of billable hours. The structure of incentive pay differs among firms, some firms require lawyers to bill at least 1,600 hours a year, others demand much higher numbers. The size of bonuses also differs among firms.

In the US, many law firms provide publicly accessible information about their compensation practices in the NALP Directory of Legal Employers.[3] Among other things, these firms publicly state whether they pay bonuses to eligible lawyers, what factors a possible bonus payment is based on, and whether a minimum exists for billable hours. Hence, law firms, by simply checking the NALP Directory of Legal Employers, have a

[3]Compensation practices for this example can be found at http://www.nalpdirectory.com/index.cfm. In a similar spirit, disclosure is often a feature of executive pay of CEOs and board of directors. In the UK, for instance, regulations that involve disclosure remuneration were implemented in 2013 (Gupta et al., 2016).

very good idea about compensation practices at their competitors, meaning that incentive schemes are publicly known. Finally, not all law firms provide the NALP Directory of Legal Employers with information about their compensation practices at the same time. Accordingly, sequential determination of compensation rules is conceivable if a firm gathers information about compensation practices at competing firms, and then chooses and discloses its own compensation practice.

To have a concrete example at hand, suppose that there is a legal battle, and the involved parties are represented by Baker & McKenzie and Shearman & Sterling, respectively, two law firms that are located in New York City. Baker & McKenzie reports in the NALP Directory of Legal Employers that both base salaries and bonuses depend on the number of billable hours and that lawyers are expected to bill at least 2,000 hours a year. In contrast, Shearman & Sterling reports that base salaries depend only on seniority and emphasizes that a minimum billable hour requirement does not exist. This means that at Baker & McKenzie firm profits (that typically depend on how successful the firm is doing in court) are more likely to be shared according to individual performance, whereas at Shearman & Sterling individual performance is relatively less important. In other words, the choice of different compensation contracts can be understood as a choice of different sharing rules.

Our results clearly have implications regarding the extent of aggregate effort expenditures. When the large group selects its sharing rule before the small one, and provided that the prize is private enough so that both groups are active, total rent-seeking expenditures increase with respect to the simultaneous case. Conversely, when the small group is the leader, and again provided that both groups are active, aggregate effort is lower compared to the simultaneous case. Thus, if from the designer's point of view effort is valuable, he should possibly oblige the two groups to choose their sharing rules simultaneously, or even force the large group to move first. If effort is instead considered as wasteful, the designer should opt for the design where the small group is the leader. In this latter case, in fact, the designer should not intervene in the game since the unique equilibrium of the game with an endogenous choice of the timing structure gives the leadership to the small group whenever both groups are active.

2 The Model

There are two groups $i = A, B$ with $n_i \in \mathbb{N}$ members and without loss of generality A is the large group (i.e., $n_A > n_B > 1$).[4] Each member $k = 1, 2, ..., n_i$ of group i chooses his individual level of effort $e_{ki} \geq 0$ whose cost is linear. The valuation of the prize is the same for all individuals, and is denoted by V. The probability that group i wins the between-group competition is given by

$$P_i = \begin{cases} \frac{1}{2} & \text{for } E_A = E_B = 0 \\ \frac{E_i}{E_A + E_B} & \text{otherwise} \end{cases}$$

where $E_i = \sum_{j=1}^{n_i} e_{ji}$ is the total effort of group i.[5] Individuals are risk neutral, and the expected utility of member $k = 1, ..., n_i$ of group i is given by

$$EU_{ki} = \begin{cases} \left\{ \left[\alpha_i \frac{e_{ki}}{E_i} + (1 - \alpha_i) \frac{1}{n_i} \right] p + (1 - p) \right\} \frac{E_i}{E_A + E_B} V - e_{ki} & \text{for } E_i > 0, E_j \geq 0 \\ \left(\frac{1}{n_i} p + 1 - p \right) \frac{1}{2} V & \text{for } E_i = E_j = 0 \\ 0 & \text{for } E_i = 0, E_j > 0 \end{cases} \tag{1}$$

for $i = A, B$ and $i \neq j$. The parameter $p \in [0, 1]$ denotes the degree of privateness of the prize ($p = 0$ corresponds to the case of a pure public good while $p = 1$ corresponds to the case of a pure private good). Indeed, an important feature of collective contests is that the prize sought by competing groups may often be interpreted as a mixture between a public and a private good. Local governments competing for funds typically devote them

[4] If $n_A = n_B$ the timing of choices of group sharing rules is irrelevant (see footnote 10).

[5] This type of contest success function is widely used in the literature and has been axiomatized by Skaperdas (1996). Alternatively, many authors have used all-pay auctions to model competition, which have been analyzed by Baye et al. (1996).

to the provision of both monetary transfers and local public goods. Similarly, prizes in research and development races involve both reputational and monetary benefits for the winning team. More generally, any prize sought by competing groups may be interpreted as a mixture between a public and a private good insofar as the winners derive some benefits in terms of status, reputation, or satisfaction following a victory.

The sharing rule α_i represents the relative weight given to meritocracy (i.e., within-group relative effort, e_{ki}/E_i) as opposed to egalitarianism in the allocation of the private part of the prize within group i.[6] If $\alpha_i > 0$ better performers in group i receive a larger share of the prize than worse performers, while if $\alpha_i = 0$ the private part of the prize is shared equally among group members regardless of their relative contributions.

Previous literature on sharing rules in collective rent seeking has considered both the cases of $\alpha_i \in [0, 1]$ (Baik, 1994; Lee, 1995; Noh, 1999; Ueda, 2002) and $\alpha_i \in [0, \infty)$ (Baik and Shogren, 1995; Baik and Lee, 1997, 2001; Lee and Kang, 1998; Balart et al., 2016). If $\alpha_i > 1$, the sharing rule of group i allows for transfers among its members, as in Hillman and Riley (1989). In such case, group i collects $-(1-\alpha_i)\frac{p}{n_i}\frac{E_i}{(E_A+E_B)}V$ from each of its members and allocates $\alpha_i p\frac{E_i}{(E_A+E_B)}V$ according to their relative contributions.[7] In this paper we analyze the sequential choice of sharing rules with and without such transfers.

Our game consists in the sequential choice of the groups' sharing rules, followed by the simultaneous choice of individuals' effort levels. The equilibrium concept is subgame perfection in pure strategies. At the effort stage we focus on within-group symmetric equilibria.[8]

[6] Observe that $1/n_i$ substitutes e_{ki}/E_i in (1) to avoid an indeterminacy for $E_i = 0$.

[7] Cost-sharing in collective contests for purely public prizes can also be interpreted in terms of within-group transfers (Nitzan and Ueda, 2014; Vazquez, 2014).

[8] As shown by Balart et al. (2016) the within group symmetric equilibrium is unique except for the specific cases where $p = 0$ or $\alpha_i = 0$, provided that group i is active. In all these cases the asymmetric equilibria yield the same aggregate effort as the symmetric one.

2.1 Effort Stage and Simultaneous Sharing Rules

Before analyzing the sequential choice of sharing rules, we describe the outcome of the last stage of the game, which is solved in Balart et al. (2016). At the effort stage, groups' members choose their level of effort by maximizing (1) subject to efforts being non-negative. The equilibrium essentially generalizes the results of Ueda (2002) and Davis and Reilly (1999) by allowing for a mixed public-private prize, thereby leading to the possibility of monopolization. More precisely, group i retires from the contest whenever

$$\alpha_i \leq \frac{n_i \left[\alpha_j p(n_j - 1) - n_j(1-p)\right] - n_j p}{(n_i - 1) n_j p} \tag{2}$$

with $i = A, B$ and $j \neq i$. If (2) holds, group i is inactive (i.e., $e_{ki} = \tilde{e}_i = 0$ for all $k \in i$) while the members of group j compete in a standard n_j-players Tullock contest for a prize of valuation $\alpha_j pV$ (i.e., the private part of the prize that is allocated according to relative effort) and thus exert effort

$$e_{kj} = \tilde{e}_j = \frac{\alpha_j p(n_j - 1)}{n_j^2} V \;\; \forall k \tag{3}$$

Conversely, if (2) is violated for $i = A, B$, then both groups are active in equilibrium and individual effort is given by

$$e_{ki} = \hat{e}_i = \frac{\Lambda_i \Phi_i}{n_i \left[n_j p + n_i(2n_j(1-p) + p)\right]^2} V \;\; \forall k, \;\; \forall i = A, B \tag{4}$$

where $\Lambda_i = [n_j(1-p) + p]\,[n_i(1-p) + p + (n_i - 1)p\alpha_i] + (n_j - 1)\,[n_i(1-p) + p]\,p\alpha_j$ and $\Phi_i = n_j p(1 - \alpha_i) + n_i[n_j(1 - p(1 - \alpha_i + \alpha_j)) + p\alpha_j]$. As can be seen from (2), the less (more) meritocratic the sharing role of group i (j), the more likely that monopolization occurs, so that group i is inactive.

The GSP arises if and only if $E_A < E_B$, which is equivalent to the small group achieving a strictly higher probability of winning than the large group. With exogenous sharing rules the GSP arises if and only if

$$\alpha_A < \frac{n_A - n_B + 2\alpha_B n_A(n_B - 1)}{2n_B(n_A - 1)} \tag{5}$$

Esteban and Ray (2001) assume that the private part of the prize is shared equally among group members regardless of their individual contribution, and find that with a linear cost of effort, the GSP arises for all values of p except for the extreme case of $p = 0$. If the prize is purely public, the group sharing rules are irrelevant and both groups win the prize with an equal probability.[9] For $p > 0$, introducing the possibility of (exogenous) group-specific sharing rules yields a more nuanced result. As can be seen from (5), for any degree of privateness of the prize, the less (more) meritocratic the sharing rule of the large (small) group, the more likely that the GSP arises.[10]

Balart et al. (2016) solve the game where the two groups choose their sharing rules simultaneously prior to the choice of individual effort, and show that the threshold $p_1 = \frac{n_B(n_A - n_B - 1)}{1 + n_B(n_A - n_B - 1)}$ determines the occurrence of monopolization: if $p \in (0, p_1]$, then only the large group (i.e., group A) is active in equilibrium, while if $p > p_1$ both groups are active. Hence, allowing for the contested prize to have both a public and a private component may lead to the inactivity of the small group. This contrasts with the cases of a purely public and purely private prize, for which monopolization never occurs.

Group size affects the groups in two different manners. As it has a greater number of potential contributors, the large group clearly has a size advantage regardless of the exact nature of the prize. However, as a smaller group size also increases the per-capita value of the private component of the prize, the small group may also enjoy a size advantage, which is greater the higher the degree of privateness of the prize. We denote by α_i^S, $i = A, B$, the equilibrium sharing rules that arise when the two groups decide them simultaneously. The implications of a simultaneous choice of sharing rules are illustrated in Figure 1: for degrees of privateness lower than p_1 the size advantage of the large group is overwhelming.

[9]This case is also equivalent to the collective contest with pure public goods studied by Baik (2008).

[10]We exclude the case of $p = 0$ as the sharing rules do not apply when the prize is a pure public good. The results for $p = 0$ can be obtained from Baik (2008).

Given that the sharing rules allow for transfers among group members, the large group selects a sharing rule that places great emphasis on relative effort (i.e., $\alpha_A^S > 1$ for all $p < p_1$), inducing high levels of effort by its members. This, combined with the fact that the large group is more numerous, discourages the small group's members from actively taking part in the competition. When the degree of privateness exceeds p_1, it becomes optimal for the small group to select a highly meritocratic sharing rule (i.e., $\alpha_B^S > 1$ for all $p > p_1$) so as to induce positive effort levels by its members. Thus, for degrees of privateness larger than p_1, both groups are active. As the degree of privateness increases further, so does the size advantage of the small group, ultimately giving rise to the GSP. In particular, when the degree of privateness exceeds the threshold $p_{GSP} = \frac{2n_A n_B}{1+2n_A n_B}$, the small group, besides being active, achieves a higher winning probability than the large group.

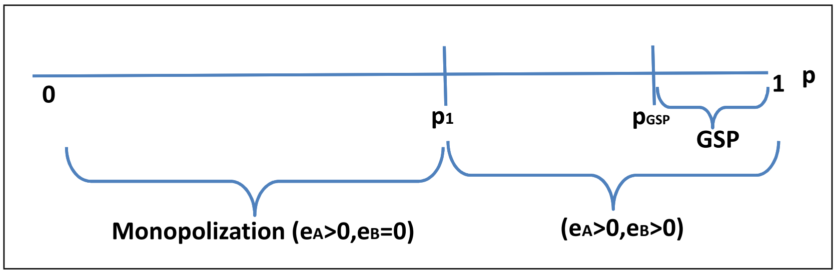

Figure 1: Simultaneous choice of sharing rules

3 Results

Before analyzing the sequential choice of sharing rules by the two groups, the following result will be useful in order to understand the way the equilibrium is altered.

Proposition 1. *If both groups are active, the sharing rules are strategic complements from the perspective of the large group, whereas they are strategic substitutes from the perspective of the small group, that is* $\frac{\partial \alpha_A}{\partial \alpha_B}(\alpha_B) > 0$ *and* $\frac{\partial \alpha_B}{\partial \alpha_A}(\alpha_A) < 0$.

This is broadly consistent with previous findings on the effects of precommitment in individual contests with asymmetric players. The sequential choice of sharing rules resembles the choice of precommitted efforts studied by Dixit (1987), who establishes that with two asymmetric players, efforts are strategic complements from the perspective of the advantaged player, while they are strategic substitutes from the perspective of the underdog.[11]

In the following subsections where the game is sequential, we consider the following three stages: In stage one the leader (group $i \in \{A, B\}$) chooses its sharing rule α_i^L. In stage two the follower (group $j \neq i$) chooses its sharing rule α_j^F. Both groups do so by maximizing their aggregate welfare (i.e., $\max \sum_{k \in i} EU_{ki}$).[12]

In stage three the members of the two groups simultaneously and individually choose their effort levels (e_i^L, e_j^F). In what follows, we assume that $p > 0$ and we present the equilibrium choices of sharing rules (α_i^L, α_j^F) in our formal results. Effort levels (e_i^L, e_j^F) chosen in stage three can be derived directly from expressions (3) and (4).

Large group A is the leader

Proposition 2. *Let* $p_1' = \frac{n_A(n_A - n_B - 1)}{1 + n_A(n_A - n_B - 1)}$ *and group A be the leader:*

- *If $p > p_1'$, both groups are active and the sharing rules in the unique subgame perfect equilibrium are given by*
  - $\alpha_A^L = 1 + n_A \frac{1-p}{p}$
  - $\alpha_B^F = \frac{n_A[n_B(1-p)+p][n_A(2n_B(1-p)-2+3p)+(n_B-2)p]-(n_A-1)(n_A-n_B)p[n_A(1-p)+p]}{2n_A(n_B-1)[n_A(1-p)+p]p}$
- *If $0 < p \leq p_1'$, only group A is active and the continuum of sharing rules in the subgame perfect equilibrium is given by*

[11] In Dixit (1987) efforts can be strategic substitutes from the perspective of the advantaged player provided his effort level is sufficiently small. In our case the strategic complementarity for the large group holds independently of the value of the sharing rule. In other words, strategic behavior in the choice of sharing rules only depends on group size. Further, if the two groups have the same size (i.e., $n_A = n_B = n$), the equilibrium sharing rules are given by $\alpha_A = \alpha_B = 1 + n\frac{1-p}{p}$ regardless of the order of moves, as in this case best responses are independent of the other group's sharing rule. This result also parallels the one in Dixit (1987), that is, when the two contestants are symmetric, cross derivatives are zero and as a consequence the timing of the game does not alter the precommitted effort levels.

[12] As the aggregate welfare of group i only depends on aggregate effort, and as in equilibrium aggregate effort is unique for any α_i, it follows that the sharing rule α_i that maximizes the expected utility of the representative individual in a within-group symmetric equilibrium also maximizes $\sum_{k \in i} EU_{ki}$.

- $\alpha_A^L = \frac{n_A}{(n_A-1)} \frac{n_B(1-p)+p}{p}$
- $\alpha_B^F \in [0, \frac{n_B(1-p)+p}{p}]$

When the large group is the leader, it prevents the small group from being active for a larger range of p than in the simultaneous case (i.e., $p_1' > p_1$). In other words, the large group acting as the leader is capable of tying the small group's hands and oblige the latter to react with a sharing rule leading to its own inactivity, and this for a larger range of p. In turn, the small group is indifferent between choosing any of the sharing rules belonging to the interval such that it retires from the contest. The remaining intuitions regarding the relative levels of meritocracy and individual efforts within each group are similar to the ones of the simultaneous case.

The following result will be useful in order to understand the implications of the sequential interaction:

Proposition 3. *When the large group A is the leader:*

- *If $p > p_1$ then aggregate effort is strictly greater than in the simultaneous case.*
- *If $p \leq p_1$ then aggregate effort is smaller than in the simultaneous case.*

When the large group has the leading advantage and both groups are active, we are in a situation that is very similar to a standard Stackelberg duopoly model. Once the large group commits to a more meritocratic rule than in the simultaneous case ($\alpha_A^L > \alpha_A^S$), the follower reacts by selecting a less meritocratic rule than in the simultaneous case ($\alpha_B^F < \alpha_B^S$). Indeed, recall that the sharing rules are strategic substitutes from the perspective of the small group (Proposition 1). Given that the large group chooses to strengthen competition between groups, these equilibrium sharing rules are such that aggregate rent-seeking expenditures are strictly greater than when the groups select their sharing rules simultaneously.

If p is such that monopolization occurs in both the simultaneous and sequential cases (i.e., $p \leq p_1$), however, the above results are reversed. That is, the large group acting as the leader reduces the level of meritocracy of its sharing rule with respect to the simultaneous case, so that aggregate effort is smaller. Indeed, being the first mover enables the large group to select its preferred equilibrium (i.e., the lowest α_A) out of the continuum arising in the simultaneous case.

Finally, recall that when $p_1 < p < p_1'$ both groups are active in the simultaneous case while only the large one is active when being the leader. Yet, the total effort that the large group's members exert in order to exclude the small group from the competition exceeds the sum of both groups' efforts in the simultaneous case.

Proposition 4. *If the large group is the leader and for all values of p including full privateness, the GSP never arises.*

Contrary to the simultaneous case, the GSP never takes place regardless of the degree of privateness of the prize, which is a direct consequence of Proposition3. Indeed, as the large group acting as the leader selects a more meritocratic sharing rule than in the simultaneous case, and as the small group reacts with a less meritocratic one, it follows that aggregate effort increases in the large group, while it decreases in the small group. As it turns out, these effort variations are large enough to eliminate the GSP.

Small group B is the leader

Proposition 5. *Let group B be the leader:*

- *If $p > p_1$, both groups are active and the sharing rules in the unique subgame perfect equilibrium are given by*
  - $\alpha_A^F = \frac{n_B[n_A(1-p)+p][n_B(2n_A(1-p)-2+3p)+(n_A-2)p]+(n_B-1)(n_A-n_B)p[n_B(1-p)+p]}{2(n_A-1)n_B[n_B(1-p)+p]p}$
  - $\alpha_B^L = 1 + n_B\frac{(1-p)}{p}$
- *If $0 < p \leq p_1$, only group A is active and the continuum of sharing rules in the subgame perfect equilibrium is given by*
  - $\alpha_A^F = \frac{n_A}{n_A-1}\left[\frac{1-p}{p} + \frac{(n_B-1)\alpha_B^L+1}{n_B}\right]$

– $\alpha_B^L \in [0, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}]$

For intermediate degrees of privateness covering most of the interval of p ($0 < p \leq p_1$), the small group is not able to take advantage of its leadership. Regardless of the choice of the small group in stage one, in stage two the large group selects a highly meritocratic sharing rule so that the small group retires from the contest in stage three. Observe that the threshold level of privateness that determines whether the small groups is active or inactive is identical to the one obtained in the simultaneous case, that is, the small group cannot take advantage of its leadership for any $p \leq p_1$. Given that the sharing rules are strategic complements for the large group, when the small group is the leader it chooses to lower the level of meritocracy of its sharing rule in order to induce the same behavior by the large group. As the small group's sharing rule is smaller than in the simultaneous case, the large group must then be able to keep the small group inactive for at least the same range of p as in the simultaneous case.

Proposition 6. *When the small group B is the leader:*

- *If $p > p_1$ then aggregate effort is strictly smaller than in the simultaneous case.*
- *If $p \leq p_1$ then total aggregate effort is strictly smaller than in the simultaneous case for $\alpha_B^L < \frac{n_B(1-p)+p}{p}$, while it can be either greater or smaller than in the simultaneous case for $\alpha_B^L \geq \frac{n_B(1-p)+p}{p}$.*

For all degrees of privateness of the prize such that both groups are active, and in stark contrast to the case in which the large group is the leader, it turns out that when the small group has the leadership advantage it strategically chooses to weaken competition by selecting a less meritocratic rule than in the simultaneous game ($\alpha_B^L < \alpha_B^S$). In turn, the large group also reacts with a less meritocratic rule than the one it selects when the game is simultaneous ($\alpha_A^F < \alpha_A^S$). Indeed, recall that the sharing rules are strategic complements from the perspective of the large group (Proposition 1). Therefore, both groups adopt relatively more egalitarian rules, their members exert a lower level of effort than in the simultaneous game $((e_A^F, e_B^L) < (e_A^S, e_B^S))$, and aggregate effort is smaller than when the groups select their sharing rules simultaneously.

As when the large group is the leader, when p is such that the small group is inactive in both the simultaneous and sequential cases (i.e., $p \leq p_1$), the above results may be reversed. That is, depending on which particular sharing rule is chosen by the small group, aggregate effort may be greater in the simultaneous than in the sequential case.

Proposition 7. *If the small group is the leader and for all values of p including full privateness, the GSP never arises.*

Observe that here, the GSP does not arise for a very different reason than when the large group is the leader. The small group selects a sharing rule such that its members are inactive when the prize is public enough (but not purely public), which clearly prevents the occurrence of the GSP as in the previous cases. Then, if $p > p_1$ the small group acting as the leader strategically chooses to weaken competition by selecting a less meritocratic rule than in the simultaneous game, which in turn induces the large group to adopt a similar behavior. However, the reduction in meritocracy adopted by the small group is relatively larger than the corresponding reduction in the large group, and is such that the GSP vanishes.

Figure 2 summarizes our findings regarding the sequential choice of sharing rules and the occurrence of monopolization and the GSP.

We have analyzed the relationship between the size of a group and its performance in terms of its probability of winning the prize. However, another relevant notion of group effectiveness relates its size to per-capita payoffs. Even though the large group always outperforms (or ties) the small one in terms of winning probability when the order of moves is sequential, the members of the small group achieve strictly higher expected utility than the members of the large group when the prize is sufficiently private. This is also the true when the choice of sharing rules is simultaneous, from a degree of privateness

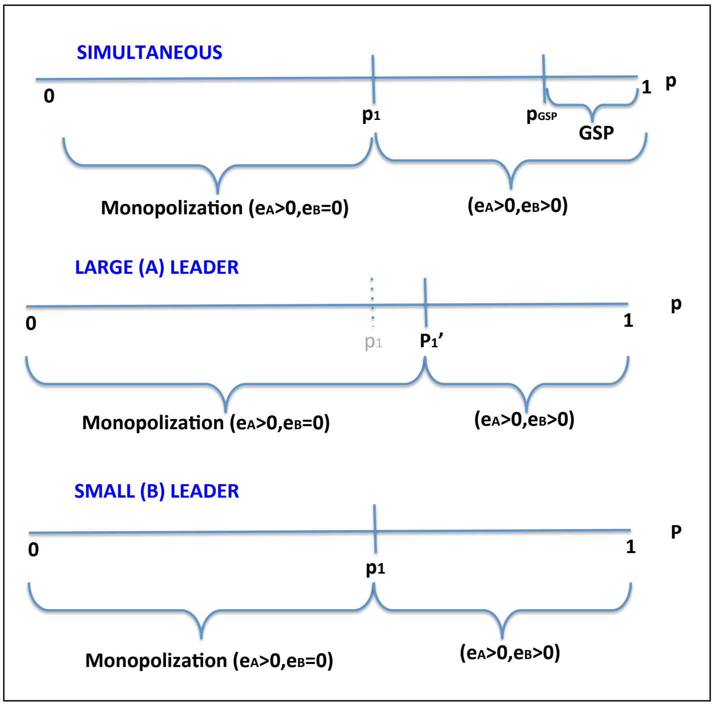

Figure 2: Simultaneous versus sequential choice of sharing rules

strictly smaller than the one required for the GSP to arise.[13] In fact, irrespective of the order of moves, when the prize is sufficiently private so that both groups are active, individual utility is decreasing in privateness in the large group, while it is increasing in the small group. With this alternative notion of group effectiveness, therefore, the members of the small group always outperform the members of the large group in a competition over a pure private good regardless of the timing of the interaction. As a larger group size reduces the per-capita value of the private component of the prize, the large group has some intrinsic disadvantage in terms of payoffs.

Although the size of the leader does not matter in terms of the GSP, it does have some implications regarding the expected utility of groups' members and aggregate effort. Consider the case where both groups are active for any timing of the interaction. When the large group is the leader, all its members clearly have higher expected utility than in the simultaneous version of the game, while all the members of the small group (the fol-

[13]The exact value of the threshold is given by $p = 1 - \left[1 + n_i \left[(\sqrt{n_A} + \sqrt{n_B})^2 - 1\right]\right]^{-1}$ when group i is the leader ($i = A, B$). Although the corresponding threshold for the simultaneous case cannot be derived analytically, one can show that it is unique given the strict monotonicity of expected utility with respect to privateness in both groups.

lower) are worse off. Therefore, a transition from simultaneous to sequential competition where the large group is the leader never consists in a Pareto improvement, and increases aggregate effort. Conversely, when the small group is the leader, the members of both groups exert less effort than in the simultaneous game, and they achieve higher expected utility. Thus, going from simultaneous to sequential competition where the small group is the leader consists in a Pareto improvement and reduces aggregate effort when $p > p_1$.

Given these implications, a natural extension of our model consists in endogenizing the timing of the strategic interaction between the two groups. To do so, let us assume that prior to the choice of sharing rules, groups are able to declare their intention to be the leader or the follower of the game.[14] Let groups have the possibility of choosing between two dates so as to declare their sharing rules. If they choose the same date, sharing rules are chosen simultaneously. If they choose different dates, sharing rules are chosen simultaneously. The results are summarized in the following proposition:

Proposition 8.

- *If $p > p_1$ the unique equilibrium of the timing game is such that the small group is the leader.*
- *If $p \leq p_1$ the payoff-dominant equilibrium of the timing game is such that the small group is the leader.*

Suppose that the prize is private enough so that both groups are active in the simultaneous setup. If the small group selects its sharing rule at date one, the large group prefers to wait, whereas if the small group selects its sharing rule at date two, the large group takes the lead. Hence the large group always prefers the sequential version of the game. On the contrary, if the large group selects its sharing rule at date one, so does the

[14] In the context of competition among individuals, several authors have considered endogenous timing extensions regarding rent-seeking expenditures (Baik and Shogren, 1992; Leininger, 1993; Morgan, 2003). The industrial organization literature has extensively considered endogenous timing extensions. See for instance Amir and Grilo (1999), Deneckere and Kovenock (1992), Hamilton and Slutsky (1990) and Van Damme and Hurkens (1999) in the context of a duopoly model.

small group, whereas if the large group moves at date two, the small group takes the lead. Therefore, the unique equilibrium of the game with an endogenous choice of the timing structure gives the leadership to the small group. This is also the case under monopolization, provided that we select the payoff-dominant equilibrium among all the possible equilibria.[15] Again, the fact that the sequential game arises endogenously constitutes a strong justification for departing from the simultaneity assumption. Further, with an endogenous timing selection previous to the rent-seeking activity, the GSP should never take place.

Sharing rules without transfers

Balart et al. (2016) show that when the sharing rules do not allow for transfers among individuals (i.e., $\alpha_i \in [0,1]$ for $i = A, B$) and their choice is simultaneous, neither monopolization nor the GSP arise. As it turns out, the equilibrium sharing rules are the same regardless of the particular timing of the game:

Proposition 9. *Let*

$$\mathring{p} = \frac{1}{2}\left\{ \frac{n_A n_B[2-5n_B+n_A(4n_B-1)]}{n_B+n_A[n_B[2-3n_B+n_A(2n_B-1)]-1]} - \sqrt{\frac{n_A n_B^2\left[8n_B+n_A\left[n_A^2+(n_B-12)n_B+2n_A(2+n_B)-4\right]\right]}{[n_B+n_A[n_B(2-3n_B+n_A(2n_B-1))-1]]^2}} \right\}$$

If the sharing rules do not allow for transfers among individuals and regardless of the order of moves:

- *If $p > \mathring{p}$, both groups are active and the sharing rules in the unique subgame perfect equilibrium are given by:*
  - $\alpha_A = \frac{n_A^2 n_B[2n_B(1-p)+p](1-p)-n_B p[2n_B(1-p)+p]-n_A\left[2n_B(1-2p)p+p^2+n_B^2(2+p(5p-7))\right]}{2(n_A-1)n_B[n_B(1-p)+p]p}$
  - $\alpha_B = 1$

[15] Given the multiplicity of equilibria when $p \leq p_1$, any timing can arise endogenously as an equilibrium depending on the particular sharing rule that group B selects out of the continuum. A reasonable selection criterion which allows a comparison across the three versions of the game is payoff dominance. Given that the small group B is inactive regardless of the particular timing (and thus obtains zero profits), payoff dominance simply requires that monopolization occurs with the small group selecting the smallest α_B out of the continuum. Further, we also use payoff dominance to select among the multiple Nash equilibria of the timing game.

- *If* $0 < p \leq \mathring{p}$, *both groups are active and the sharing rules in the unique subgame perfect equilibrium are given by* $\alpha_A = \alpha_B = 1$.

As the constraints on the sharing rules are binding for the same range of parameters regardless of the timing of the interaction, allowing one group to choose its sharing rule first does not alter the strength of competition compared to the simultaneous case. Further, in the absence of transfers, neither the large nor the small group can select a sharing rule that enables them to fully exploit their size advantage. As a consequence, neither monopolization nor the GSP occurs. In other words, the large (small) group needs to implement transfers among its members in order to fully exploit their respective size advantages.

4 Discussion

Collective contests have drawn a lot of attention given their wide range of applications. In this paper, we contribute to this literature by studying the case in which groups are allowed to determine their sharing rules sequentially. We believe such an analysis is relevant for several reasons. First, we show that the sequential choice of sharing rules emerges endogenously when the groups are allowed to decide upon the timing of the game. Second, we provide real life examples in which the compensations of organizations' members are public information and are chosen in a sequential fashion. Thus, our framework fits well with compensation schemes in litigation firms or with the debate on the regulation of executive compensation disclosure. Third, we show that the timing according to which organizations of different sizes choose their sharing rules has several consequences of interest from the perspective a contest designer. Despite being highly stylized, our model provides interesting insights regarding the interaction between groups size, the nature of the prize and the timing of choices.

The relative size of the first mover is key to our results. The large group is more aggressive when allowed to move first than in the simultaneous contest, while the small group acting as the leader follows the opposite strategy. This is a direct consequence of the fact that group sharing rules are strategic complements from the perspective of the large group, while they are strategic substitutes from the perspective of the small group. When sharing rules allow for within-group transfers, the public-private nature of the prize plays an important role. When the large group moves first and organizations compete for a prize that is close to the definition of a pure private good, aggregate effort is greater than in the simultaneous case. However, the opposite is true when the public component of the contested-prize is sufficiently high, or when the small group moves first and the contested prize is sufficiently private. In contrast, the particular timing of the game is irrelevant when sharing rules do not allow for within group transfers.

We also provide interesting insights regarding the GSP and monopolization, two situations that can arise when the choice of sharing rules is simultaneous and provided they allow for transfers among group members. First, the GSP never arises when the choice of sharing rules is sequential, regardless of the leader's size. When allowed to move first, the large group increases the degree of meritocracy of its sharing rule, thereby increasing the effort of its members and preventing the small group to outperform. On the contrary, the small group acting as the leader does exactly the opposite, which reduces its members' effort thus making it easier for the large group to outperform. Regarding the occurrence of monopolization, the large group takes advantage of its leadership by excluding the small group from the competition for a larger set of parameters than in the simultaneous case. Yet, this is not the case for the small group, as having a first mover advantage does not allow its members to reduce the set of parameters for which monopolization occurs.

5 Appendix

Preliminaries

Before proceeding with the proofs of our results we present the relevant results of Balart et al. (2016) regarding groups' best responses and the equilibrium sharing rules in the simultaneous game.

Given the equilibrium effort levels if both groups are active as presented in equation (4), the expected utility for the representative individual in group $i = A, B$ is given by

$$\hat{EU}_i(\alpha_A, \alpha_B) = \frac{[n_i n_j + ((\alpha_i - 1)(n_i - 1)n_j + \alpha_j n_i(1 - n_j))p]V}{n_i[n_j p + n_i(2n_j(1-p) + p)]^2} [n_i(2n_i - 1)n_j - Ap + B(n_i - 1)p^2]$$

where

$$A = n_i(1 - \alpha_j - n_i) + n_j(1 - \alpha_i) + n_i(4n_i + \alpha_i + \alpha_j - 5)n_j$$

$$B = \alpha_i(n_j - 1) + \alpha_j(n_j - 1) + (n_i - 1)(2n_j - 1)$$

Given the equilibrium effort levels if only group i is active as presented in equation (3), the expected utility for the representative individual in group i is given by

$$\tilde{EU}_i(\alpha_i) = V - \frac{(n_i - 1)(n_i + \alpha_i)}{n_i^2} pV$$

Let $\chi_i(\alpha_A, \alpha_B) = (1 - \alpha_i)p(n_i n_j - n_j) - (1 - \alpha_j)p(n_i n_j - n_i) - n_i n_j(1 - p) - pn_i$ with $i = A, B$ and $j \neq i$. Condition (2) for the occurrence of monopolization is equivalent to $\chi_i(\alpha_i, \alpha_j) \geqslant 0$. Notice that if $\chi_i(\alpha_i, \alpha_j) \geqslant 0$ then $\chi_j(\alpha_i, \alpha_j) < 0$, meaning that if i is inactive j must be active.

Solving $\chi_j(\alpha_{i2}(\alpha_j), \alpha_j) = 0$ we define $\alpha_{i2}(\alpha_j) = \frac{n_i}{n_i - 1}\left[\frac{1-p}{p} + \frac{(n_j - 1)\alpha_j + 1}{n_j}\right]$ as the minimum value of α_i that guarantees that group j's members are inactive in equilibrium given any α_j.

Solving $\chi_i(\alpha_{i3}(\alpha_j), \alpha_j) = 0$ we define $\alpha_{i3}(\alpha_j) = \frac{n_j p + n_i[n_j + \alpha_j p - (\alpha_j + 1)n_j p]}{(1 - n_i)n_j p}$ as the maximum value of α_i that guarantees that group i's members are inactive in equilibrium given any α_j. This means that for all $\alpha_i \in [0, \alpha_{i3}(\alpha_j)]$ group i is inactive.

As $\alpha_{i2}(\alpha_j) > \alpha_{i3}(\alpha_j)$ we can write group i's expected utility as follows:

$$EU_i(\alpha_i) = \begin{cases} 0 & \text{if } 0 \leq \alpha_i \leq \alpha_{i3}(\alpha_j) \\ \hat{EU}_i(\alpha_A, \alpha_B) & \text{if } \alpha_{i3}(\alpha_j) < \alpha_i < \alpha_{i2}(\alpha_j) \\ \tilde{EU}_i(\alpha_i) & \text{if } \alpha_i \geq \alpha_{i2}(\alpha_j) \end{cases}$$

$EU_i(\alpha_i)$ is a continuous function and $\hat{EU}_i(\alpha_A, \alpha_B)$ is a strictly concave function (with an unrestricted domain) that reaches a global maximum at

$$\alpha_{i1}(\alpha_j) = \frac{n_j[n_i(1-p)+p][n_j(2-2n_i(1-p)-3p)-(n_i-2)p]-(n_j-1)(n_i-n_j)p^2\alpha_j}{2(n_i-1)n_j[n_j(p-1)-p]p}$$

Using their analytical expressions, we have that $\alpha_{i1}(\alpha_j) \leq \alpha_{i3}(\alpha_j)$ if and only if

$$\alpha_j \geq \frac{n_j}{n_j - 1}\frac{n_i(1-p)+p}{p} = \hat{\alpha}_j$$

and $\alpha_{i1}(\alpha_j) \geq \alpha_{i2}(\alpha_j)$ if and only if

$$\alpha_j \leq \frac{n_j[n_i(1-p)+3p-2]-2p}{(n_j-1)p} = \tilde{\alpha}_j$$

Using the above thresholds, the best response of group i can be summarized as

$$\alpha_i(\alpha_j) = \begin{cases} \alpha_{i2}(\alpha_j) & \text{for } \alpha_j \leq \tilde{\alpha}_j \\ \alpha_{i1}(\alpha_j) & \text{for } \tilde{\alpha}_j < \alpha_j < \hat{\alpha}_j \\ \dot{\alpha}_i \in [0, \alpha_{i3}(\alpha_j)] & \text{for } \alpha_j \geq \hat{\alpha}_j \end{cases}$$

If group j selects a very "meritocratic" rule (i.e., $\alpha_j > \hat{\alpha}_j$) then it is not worth for group i to be active, so that it selects $\dot{\alpha}_i$. If on the contrary group j selects a very "egalitarian" rule (i.e., $\alpha_j \leq \tilde{\alpha}_j$), it is then group i that prevents group j from being active. For intermediate values of α_j, group i selects $\alpha_{i1}(\alpha_j)$ and thus both groups are active.

Using the above reaction function, Balart et al. (2016) show that if sharing rules allow for transfers among group members and groups choose their sharing rules simultaneously, there exists a threshold $p_1 = \frac{n_B(n_A-n_B-1)}{1+n_B(n_A-n_B-1)}$ determining the occurrence of monopolization:

- If $p > p_1$, both groups are active and the sharing rule for $i = A, B$ in the unique subgame perfect equilibrium is given by

$$\alpha_i^S = \frac{n_i}{(n_i-1)} \frac{2n_i n_j (n_i - 1) + A_i p + B_i p^2}{p\,[2n_i n_j - p(n_i(2n_j-1) - n_j)]}$$

where $A_i = n_i n_j(9 - 4n_i) - n_j(n_j + 2) - 2n_i$ and $B_i = n_i n_j(2n_i - 7) + n_j(n_j + 3) + 3n_i - 2$.

- If $0 < p \leq p_1$, only group A is active and the continuum of sharing rules in the subgame perfect equilibrium is given by
  - $\alpha_A^S = \frac{n_A}{n_A-1}\left[\frac{1-p}{p} + \frac{(n_B-1)\alpha_B+1}{n_B}\right]$
  - $\alpha_B^S \in [\frac{n_B(1-p)+p}{p}, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}]$

Proof of Proposition 1. Taking derivatives of $\alpha_{A1}(\alpha_B)$ and $\alpha_{B1}(\alpha_B)$ we directly obtain the result:
$\frac{\partial \alpha_{A1}(\alpha_B)}{\partial \alpha_B} = \frac{(n_B-1)(n_A-n_B)p}{2(n_A-1)n_B[n_B(1-p)+p]} > 0$, hence α_B is a strategic complement for group A.
$\frac{\partial \alpha_{B1}(\alpha_A)}{\partial \alpha_A} = -\frac{(n_A-1)(n_A-n_B)p}{2n_A(n_B-1)[n_A(1-p)+p]} < 0$, hence α_A is a strategic substitute for group B.

□

Proof of Proposition 2. Maximizing the expected utility of the representative individual of group i in the within-group symmetric equilibrium also maximizes the aggregate welfare of group i. Therefore, in this proof and the subsequent ones, we focus on the sharing rule α_i that maximizes the expected utility of the representative individual in group i, which we denote by $EU_i(\alpha_A, \alpha_B)$.

The best response of group B can be obtained from the simultaneous case:

$$
\alpha_B(\alpha_A) = \begin{cases} \alpha_{B2}(\alpha_A) & \text{for } \alpha_A \leq \tilde{\alpha}_A \\ \alpha_{B1}(\alpha_A) & \text{for } \tilde{\alpha}_A < \alpha_A < \hat{\alpha}_A \\ \dot{\alpha}_B \in [0, \alpha_{B3}(\alpha_A)] & \text{for } \alpha_A \geq \hat{\alpha}_A \end{cases}
$$

The expected utility of group A being the leader is given by:

$$
EU_A(\alpha_A) = \begin{cases} 0 & \text{if } \alpha_A \leq \tilde{\alpha}_A \\ \hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A)) & \text{if } \tilde{\alpha}_A < \alpha_A < \hat{\alpha}_A \\ \tilde{EU}_A(\alpha_A) & \text{if } \alpha_A \geq \hat{\alpha}_A \end{cases}
$$

Notice that $EU_A(\alpha_A)$ is a continuous function since

$$
\lim_{\alpha_A \to \tilde{\alpha}_A} \hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A)) = 0 \text{ and } \lim_{\alpha_A \to \hat{\alpha}_A} \hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A)) = \tilde{EU}_A(\hat{\alpha}_A)
$$

Moreover, the second derivative of $\hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A))$ is $\frac{[p(n_A-1)]^2 V}{2n_A^2[n_A(p-1)-p]} < 0$. Thus, $\hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A))$ is a strictly concave function in the unrestricted domain $\alpha_A \in (-\infty, +\infty)$, and obtains a global maximum at $\alpha_A^L = 1 + n_A \frac{1-p}{p}$, where α_A^L is the solution of the first-order condition $\frac{\partial \hat{EU}_A(\alpha_A, \alpha_{B1}(\alpha_A))}{\partial \alpha_A} = 0$. Given that $\tilde{EU}_A(\alpha_A)$ is strictly decreasing with respect to α_A, $\hat{\alpha}_A$ strictly dominates any $\alpha_A > \hat{\alpha}_A$. As $\alpha_A^L > \tilde{\alpha}_A$, it follows that $EU_A(\alpha_A)$ is maximized at $\min\{\alpha_A^L, \hat{\alpha}_A\}$ and thus the corresponding expected utility is strictly positive. Observe then that $\alpha_A^L < \hat{\alpha}_A$ if and only if $p > \frac{n_A(n_A-n_B-1)}{1+n_A(n_A-n_B-1)} = p_1'$. Therefore,

- If $p \leq p_1'$, group A selects $\hat{\alpha}_A = \frac{n_A}{(n_A-1)} \frac{n_B(1-p)+p}{p}$ leading to the inactivity of group B, and group B selects any $\alpha_B \in [0, \frac{n_B(1-p)+p}{p}]$, with the upper bound being the solution of $\alpha_B = \alpha_{B3}(\hat{\alpha}_A)$ guaranteeing the inactivity of group B.

- If $p > p_1'$ there exists a unique equilibrium such that both groups are active. Group A selects $\alpha_A^L = 1 + n_A \frac{1-p}{p}$ and group B selects $\alpha_{B1}(\alpha_A^L)$, given by

$$
\alpha_{B1}(\alpha_A^L) = \frac{n_A[n_B(1-p)+p][n_A(2n_B(1-p)-2+3p)+(n_B-2)p]-(n_A-1)(n_A-n_B)p[n_A(1-p)+p]}{2n_A(n_B-1)[n_A(1-p)+p]p} = \alpha_B^F
$$

□

Proof of Proposition 3.

- If $p \leq p_1$ monopolization arises both in the simultaneous and in the sequential cases and there is a continuum of equilibria. By comparing the equilibrium supports of α_B^S and α_B^F we can see that the lower bound of the former is equal to the upper bound of the latter which guarantees that $\alpha_B^F \leq \alpha_B^S$. Similarly, α_A^L is equal to the lowest possible value in the support of α_A^S, which guarantees that $\alpha_A^L \leq \alpha_A^S$. Given that only group A is active, equilibrium aggregate effort is equal to $n_A \frac{\alpha_A p(n_A-1)}{n_A^2} V$, which is strictly increasing in α_A, hence aggregate effort is greater in the simultaneous case.

- If $p_1 < p \leq p_1'$ monopolization only arises in the sequential case. Equilibrium aggregate effort in the simultaneous case where both groups are active is given by

$$E_A^S + E_B^S = \frac{n_A n_B(1-n_A-n_B)+\left[n_A+n_B+2(n_A-3)n_A n_B+2n_A n_B^2\right]p-[2n_B-1+n_A(2+n_B(n_A+n_B-5))]p^2}{n_A[2n_B(p-1)-p]-n_B p} V$$

  while equilibrium aggregate effort in the sequential case where group B is inactive is given by $E_A^L = (n_B+p-n_B p)V$. Therefore, equilibrium aggregate effort is greater in the simultaneous case if and only if $\frac{[n_A(p-1)-p][(n_A-n_B-1)n_B(1-p)-p]V}{n_A[2n_B(1-p)+p]+n_B p} \leq 0$. Notice that the denominator is always positive and the first term of the nominator is always negative. The second term of the numerator is positive if and only if $p \leq p_1$ which is not in the analyzed interval. Hence, equilibrium aggregate effort is greater in the sequential case.

- If $p > p_1'$ both groups are active in both the simultaneous and sequential cases. Thus we now have that equilibrium aggregate effort in the sequential case where the large group is the leader is given by $E_A^L + E_B^F = \frac{n_A(n_A+n_B-1)-p-n_A(n_A+n_B-3)p}{2n_A} V$. Therefore, equilibrium aggregate effort is greater in the sequential case if and only if $\frac{(n_A-n_B)[n_A(1+n_A-n_B)(1-p)+p]pV}{2n_A[n_A(2n_B(1-p)+p)+n_B p]} > 0$ which is always true for any $p > 0$.

□

Proof of Proposition 4. Clearly, if the small group is inactive, the GSP cannot occur. If $p > p_1'$, the GSP arises if and only if

$$\alpha_A^L < \frac{n_A - n_B + 2\alpha_B^F n_A(n_B - 1)}{2n_B(n_A - 1)}$$

Substituting for the equilibrium value of α_A^L and α_B^F from Proposition 1, the above condition reduces to

$$\frac{n_A(n_A - n_B)(p-1)\left[n_A(2n_B(p-1)-p) - n_B p\right]}{2(n_A - 1)n_B p\left[n_A(1-p)+p\right]} < 0$$

As the denominator is positive, the condition holds if and only if the numerator is negative, which requires $p > 1$. Hence we reach a contradiction.

□

Proof of Proposition 5. The best response of group A can be obtained from the simultaneous case:

$$\alpha_A(\alpha_B) = \begin{cases} \alpha_{A2}(\alpha_B) & \text{for } \alpha_B \leq \tilde{\alpha}_B \\ \alpha_{A1}(\alpha_B) & \text{for } \tilde{\alpha}_B < \alpha_B < \hat{\alpha}_B \\ \dot{\alpha}_A \in [0, \alpha_{A3}(\alpha_B)] & \text{for } \alpha_B \geq \hat{\alpha}_B \end{cases}$$

The expected utility of group B being the leader is given by:

$$EU_B(\alpha_B) = \begin{cases} 0 & \text{if } \alpha_B \leq \tilde{\alpha}_B \\ \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B) & \text{if } \tilde{\alpha}_B < \alpha_B < \hat{\alpha}_B \\ \tilde{EU}_B(\alpha_B) & \text{if } \alpha_B \geq \hat{\alpha}_B \end{cases}$$

Notice that $EU_B(\alpha_B)$ is a continuous function since

$\lim_{\alpha_B \to \tilde{\alpha}_B} \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B) = 0$ and $\lim_{\alpha_B \to \hat{\alpha}_B} \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B) = \tilde{EU}_B(\hat{\alpha}_B)$

Moreover, the second derivative of $\hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B)$ is equal to $\frac{[p(n_B-1)]^2 V}{2n_B^2[n_B(p-1)-p]} < 0$. Thus, $\hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B)$ is a strictly concave function in the unrestricted domain $\alpha_B \in (-\infty, +\infty)$, and obtains a global maximum at $\alpha_B^L = 1 + n_B \frac{1-p}{p}$, where α_B^L is the solution of the first-order condition $\frac{\partial \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B)}{\partial \alpha_B} = 0$. Given that $\tilde{EU}_B(\alpha_B)$ is strictly decreasing with respect to α_B, $\hat{\alpha}_B$ strictly dominates any $\alpha_B > \hat{\alpha}_B$. Observe that $\alpha_B^L < \hat{\alpha}_B$ always holds, while $\alpha_B^L \leq \tilde{\alpha}_B$ if and only if $p \leq p_1$. Therefore,

- If $p \leq p_1$, group B selects any $\alpha_B \in \left[0, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}\right]$ so that it remains inactive, and group A selects $\alpha_{A2}(\alpha_B) = \frac{n_A}{n_A-1}\left[\frac{1-p}{p} + \frac{(n_B-1)\alpha_B+1}{n_B}\right]$.
- If $p > p_1$ there exists a unique equilibrium such that both groups are active. Group B selects $\alpha_B^L = 1 + n_B\frac{(1-p)}{p}$ and group A selects $\alpha_{A1}(\alpha_B^L)$, given by

$$\alpha_{A1}(\alpha_B^L) = \frac{n_B[n_A(1-p)+p][n_B(2n_A(1-p)-2+3p)+(n_A-2)p]+(n_B-1)(n_A-n_B)p[n_B(1-p)+p]}{2(n_A-1)n_B[n_B(1-p)+p]p} = \alpha_A^F$$

□

Proof of Proposition 6.

- If $p \leq p_1$ monopolization arises both in the simultaneous and in the sequential cases and there is a continuum of equilibria. By comparing the equilibrium supports of $\alpha_B^L \in [0, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}]$ and $\alpha_B^S \in [\frac{n_B(1-p)+p}{p}, \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}]$ we can see that their upper bounds coincide. Therefore, $\alpha_B^L < \alpha_B^S$ for all $\alpha_B^L < \frac{n_B(1-p)+p}{p}$. As under monopolization group A's equilibrium sharing rule is strictly increasing in α_B, it follows that $\alpha_B^L < \alpha_B^S$ implies $\alpha_A^F < \alpha_A^S$. Given that only group A is active, equilibrium aggregate effort is equal to $n_A\frac{\alpha_A p(n_A-1)}{n_A^2}V$ which is strictly increasing in α_A, hence aggregate effort is strictly greater in the simultaneous case for any $p \leq p_1$ and $\alpha_B^L < \frac{n_B(1-p)+p}{p}$.

  When the supports of the two equilibria overlap (i.e., for $\alpha_B^L \geq \frac{n_B(1-p)+p}{p}$), α_B^L can be either greater or smaller than α_B^S. Following the same reasoning as before, we have that aggregate effort in the sequential case can be either greater or smaller than in the simultaneous case depending on the specific value of α_B one considers in each game.

- If $p > p_1$ both groups are active in both the simultaneous and sequential cases. Equilibrium aggregate effort in the simultaneous case is given by

$$E_A^S + E_B^S = \frac{n_A n_B(1-n_A-n_B)+\left[n_A+n_B+2(n_A-3)n_A n_B+2n_A n_B^2\right]p-[2n_B-1+n_A(2+n_B(n_A+n_B-5))]p^2}{n_A[2n_B(p-1)-p]-n_B p}V$$

while equilibrium aggregate effort in the sequential case where the small group is the leader is given by $E_A^F + E_B^L = \frac{n_B(n_A+n_B-1)-p-n_B(n_A+n_B-3)p}{2n_B}V$. Therefore, equilibrium aggregate effort is strictly greater in the simultaneous case if and only if $\frac{(n_A-n_B)p[(n_A-n_B-1)n_B(p-1)+p]V}{2n_B[n_A(2n_B(1-p)+p)+n_B p]} > 0$. Given that the denominator is positive, this requires that the numerator is also positive, hence that $[(n_A-n_B-1)n_B(p-1)+p] > 0$, which is true if and only if $p > p_1$. Consequently, aggregate effort in the sequential case is strictly smaller than in the simultaneous case for any $p > p_1$.

□

Proof of Proposition 7. Clearly, if the small group is inactive, the GSP cannot occur. If $p > p_1$, the GSP arises if and only if

$$\alpha_A^F < \frac{n_A - n_B + 2\alpha_B^L n_A(n_B-1)}{2n_B(n_A-1)}$$

Substituting for the equilibrium value of α_A^F and α_B^L from Proposition 3, the above condition reduces to

$$\frac{(n_A-n_B)(p-1)\left[n_A(2n_B(p-1)-p)-n_B p\right]}{2(n_A-1)p\left[n_B(1-p)+p\right]} < 0$$

As the denominator is positive, the condition holds if and only if the numerator is negative, which requires $p > 1$. Hence we reach a contradiction.

□

Proof of Proposition 8.

- $p \leq p_1$

We are in the case where monopolization occurs regardless of the particular timing of the game. As we are looking for the payoff-dominant equilibrium of the timing game, we shall assume that group B selects the minimum sharing rule out of the continuum of equilibria for each possible timing. Then we have that under the simultaneous case

$$\alpha_A^S = \frac{n_A[n_B(1-p)+p]}{(n_A-1)p} \text{ and } \alpha_B^S = \frac{n_B(1-p)+p}{p}$$

If the large group A is the leader we have

$$\alpha_A^L = \frac{n_A[n_B(1-p)+p]}{(n_A-1)p} \text{ and } \alpha_B^F = 0$$

If the large group B is the leader we have

$$\alpha_A^F = \frac{n_A[n_B(1-p)+p]}{(n_A-1)n_B p} \text{ and } \alpha_B^L = 0$$

Comparing utility levels across the different timings for the two groups yields

$$EU_A^S = EU_A^L \text{ and } EU_A^S - EU_A^F = \frac{(n_B-1)[n_B(p-1)-p]V}{n_A n_B} < 0$$

$$EU_B^S = EU_B^L = E_B^F = 0$$

Let (i,j) denote a Nash equilibria of the timing game, where i (j) is the action of group A (B), that is, $i,j = L,F$. Given the above, there will be three Nash equilibria of the timing game: (L,F),(F,L) and (F,F). Out of these three equilibria, the one where group A achieves the highest payoff is (F,L) (i.e., B is the leader). As we assumed that group B selects the smallest α_B out of the equilibrium support for any possible timing, the equilibrium (F,L) is clearly the payoff-dominant one (i.e., the one such that A achieves the highest payoff).

- $p_1 < p \leq p_1'$

We are in the case where monopolization occurs when the large group A is the leader. Equilibrium sharing rules under the simultaneous game are given by

$$\alpha_i^S = \frac{n_i}{(n_i-1)} \frac{2n_i n_j(n_i-1) + A_i p + B_i p^2}{p\left[2n_i n_j - p(n_i(2n_j-1) - n_j)\right]}$$

where $A_i = n_i n_j(9-4n_i) - n_j(n_j+2) - 2n_i$ and $B_i = n_i n_j(2n_i - 7) + n_j(n_j+3) + 3n_i - 2$. If the large group A is the leader, we know that $\alpha_A^L = \frac{n_A[n_B(1-p)+p]}{(n_A-1)p}$ for any value of α_B^F. Finally, if the small group B is the leader, we have

$$\alpha_A^F = \frac{n_B\left[n_A(1-p)+p\right]\left[n_B(2-2n_A(1-p)-3p)-(n_A-2)p\right]-(n_B-1)(n_A-n_B)p\left[n_B(1-p)+p\right]}{2(n_A-1)n_B\left[n_B(p-1)-p\right]p}$$

$$\alpha_B^L = \frac{n_B(1-p)+p}{p}$$

Given that $\alpha_i^L \neq \alpha_i^S$ for $i = A, B$, it follows directly that $EU_i^L > EU_i^S$ for $i = A, B$. We then have that

$$EU_A^S - EU_A^F = \frac{\left\{\left[n_B(n_B-n_A-1)(p-1)+p\right]^2 - \frac{4n_B^2\left[n_A(1+n_A-n_B)(1-p)+p\right]^2(n_B+p-n_Bp)^2}{\left[n_Bp+n_A(2n_B(1-p)+p)\right]^2}\right\}V}{4n_An_B\left[n_B(p-1)-p\right]}$$

Isolating p in the previous expression, $EU_A^S - EU_A^F \geq 0$ can be written as $p \leq p_1$, hence $EU_A^S < EU_A^F$ for any $p > p_1$.

$$EU_B^S - EU_B^F = \frac{n_A\left[n_A(1-p)+p\right]\left[(n_A-n_B-1)n_B(p-1)+p\right]^2 V}{n_B\left[n_Bp+n_A(2n_B(1-p)+p)\right]^2} > 0$$

Given the above, the unique Nash equilibrium of the timing game is (F, L), that is, B is the leader.

- $p > p_1'$

We are in the case where both groups are active regardless of the particular timing of the game. With respect to the previous case where $p \in (p_1, p_1']$, the only difference is that if the large group A is the leader, we now have

$$\alpha_A^L = \frac{n_A(1-p)+p}{p}$$

$$\alpha_B^F = \frac{n_A[n_B(1-p)+p][n_A(2n_B(1-p)-2+3p)+(n_B-2)p]-(n_A-1)(n_A-n_B)p[n_A(1-p)+p]}{2n_A(n_B-1)[n_A(1-p)+p]p}$$

Given that $\alpha_i^L \neq \alpha_i^S$ for $i = A, B$, it follows directly that $EU_i^L > EU_i^S$ for $i = A, B$.

We then have that

$$EU_A^S - EU_A^F = -\frac{\left\{ [n_B(n_B - n_A - 1)(p-1) + p]^2 - \frac{4n_B^2[n_A(1+n_A-n_B)(1-p)+p]^2(n_B+p-n_Bp)^2}{[n_Bp+n_A(2n_B(1-p)+p)]^2} \right\} V}{4n_An_B\left[n_B(1-p)+p\right]}$$

Isolating p in the previous expression, $EU_A^S - EU_A^F \geq 0$ can be written as $p \leq p_1$. Given that $p_1 < p_1'$, it holds that $EU_A^S < EU_A^F$ for any $p > p_1'$. Finally, we have that

$$EU_B^S - EU_B^F = -\frac{\left\{ [n_A(n_A - n_B - 1)(p-1) + p]^2 - \frac{4n_A^2[(n_A-n_B-1)n_B(p-1)+p]^2(n_A+p-n_Ap)^2}{[n_Bp+n_A(2n_B(1-p)+p)]^2} \right\} V}{4n_An_B\left[n_A(1-p)+p\right]}$$

Isolating p in the previous expression, $EU_B^S - EU_B^F \leq 0$ can be written as

$$p \leq \frac{1}{2}\left\{ \frac{n_A[n_B(5+3n_B)+n_A^2(8n_B-1)+n_A(3-2n_B(5+4n_B))]}{n_A[n_B(5+3n_B)-3+n_A^2(4n_B-1)+n_A(3-2n_B(3+2n_B))]-n_B} - \sqrt{\frac{n_A^2(n_A-n_B)^2[9+n_A^2+6n_A(n_B-1)+n_B(14+9n_B)]}{[n_B+n_A(3+(n_A-3)n_A-5n_B+2(3-2n_A)n_An_B+(4n_A-3)n_B^2)]^2}} \right\} = p^M$$

As $p^M < p_1'$, it must be the case that $EU_B^S > EU_B^F$ for any $p > p_1'$.

Given the above, the unique Nash equilibrium of the timing game is (F, L), that is, B is the leader. □

Proof of Proposition 9. The proof of the proposition for the simultaneous case can be found in Balart et al. (2016).

Large group A is the leader

We proceed by backward induction solving first the best response of the follower, i.e., the small group B. It is immediate that $\hat{\alpha}_A > 1$ and $\alpha_{B1}(\alpha_A) > 1$ for all $\alpha_A \in [0,1]$, hence without transfers the best response of B boils down to $\min\{\alpha_{B2}(\alpha_A), 1\}$, which can be written as

$$\alpha_B(\alpha_A) = \begin{cases} \alpha_{B2}(\alpha_A) & \text{if } p > \frac{n_An_B}{2n_An_B - n_A - n_B} \text{ and } \alpha_A \leq \frac{(2p-1)n_An_B - p(n_A+n_B)}{n_Bp(n_A-1)} \equiv \bar{\alpha}_A \\ 1 & \text{otherwise} \end{cases}$$

Given B's best response, group A maximizes its expected utility given by

$$EU_A(\alpha_A) = \begin{cases} 0 & \text{if } p > \frac{n_A n_B}{2n_A n_B - n_A - n_B} \text{ and } \alpha_A \leq \bar{\alpha} \\ \hat{EU}_A(\alpha_A, 1) & \text{otherwise} \end{cases}$$

Note that $\lim_{\alpha_A \to \bar{\alpha}_A} \hat{EU}_A(\alpha_A, 1) = 0$, hence $EU_A(\alpha_A)$ is a continuous function independently of the value of p. Moreover, the second derivative of $\hat{EU}_A(\alpha_A, 1)$ with respect to α_A is equal to $-\frac{2(n_A-1)^2 n_B [n_B(1-p)+p]p^2 V}{n_A [n_B p + n_A(2n_B(1-p)+p)]^2} < 0$, and thus $\hat{EU}_A(\alpha_A, 1)$ is a strictly concave function that reaches a global maximum at

$$\min\{\frac{n_A^2 n_B [2n_B(1-p)+p](1-p) - n_B p[2n_B(1-p)+p] - n_A\left[2n_B(1-2p)p + p^2 + n_B^2(2+p(5p-7))\right]}{2(n_A-1)n_B[n_B(1-p)+p]p}, 1\}$$

where the left expression is the sharing rule α_A that solves $\frac{\partial \hat{EU}_A(\alpha_A, 1)}{\partial \alpha_A} = 0$. We can rewrite

$$\frac{n_A^2 n_B [2n_B(1-p)+p](1-p) - n_B p[2n_B(1-p)+p] - n_A\left[2n_B(1-2p)p + p^2 + n_B^2(2+p(5p-7))\right]}{2(n_A-1)n_B[n_B(1-p)+p]p} < 1$$

as[16]

$$p > \mathring{p} = \frac{1}{2}\left\{\frac{n_A n_B [2 - 5n_B + n_A(4n_B - 1)]}{n_B + n_A[n_B[2 - 3n_B + n_A(2n_B - 1)] - 1]} - \sqrt{\frac{n_A n_B^2 \left[8n_B + n_A\left[n_A^2 + (n_B - 12)n_B + 2n_A(2 + n_B) - 4\right]\right]}{[n_B + n_A[n_B(2 - 3n_B + n_A(2n_B - 1)) - 1]]^2}}\right\}$$

Therefore, at the unique equilibrium we have

- $\alpha_A = \alpha_B = 1$ for $p \in (0, \mathring{p}]$
- $\alpha_A = \frac{n_A^2 n_B [2n_B(1-p)+p](1-p) - n_B p[2n_B(1-p)+p] - n_A\left[2n_B(1-2p)p + p^2 + n_B^2(2+p(5p-7))\right]}{2(n_A-1)n_B[n_B(1-p)+p]p}$ and $\alpha_B = 1$ for $p \in (\mathring{p}, 1]$

Substituting these equilibrium values in condition (2), we find that in equilibrium both groups are active.

Small group B is the leader

We can obtain group A's best response with restricted sharing rules by adding the constraint $\alpha_A \leq 1$ in the best response with transfers. It can be shown that $\alpha_{A1}(\alpha_B)$ is

[16] There are two values of p that satisfy the previous inequality. However we can disregard one of them as it is out of the interval $(0, 1]$.

chosen over $\alpha_{A2}(\alpha_B)$ if and only if $\alpha_{A1}(\alpha_B) < \alpha_{A2}(\alpha_B)$ (see the proof of Proposition 3 in Balart et al. (2016) for the details). Therefore, the best response of group A is given by

$$\alpha_A(\alpha_B) = \min\{\alpha_{A1}(\alpha_B), \alpha_{A2}(\alpha_B), 1\}$$

which can be written as

$$\alpha_A(\alpha_B) = \begin{cases} \alpha_{A2}(\alpha_B) & \text{if } p \in [p_C, p_X] \text{ and } \alpha_B \leq \alpha_2, \text{ or} \\ & \text{if } p > p_X \text{ and } \alpha_B \leq \tilde{\alpha}_B \\ \alpha_{A1}(\alpha_B) & \text{if } p \in (p_X, \mathring{p}) \text{ and } \tilde{\alpha}_B < \alpha_B \leq \alpha_1, \text{ or} \\ & \text{if } p > \mathring{p} \text{ and } \alpha_B \geq \tilde{\alpha}_B \\ 1 & \text{otherwise} \end{cases}$$

where $p_A = \frac{n_A[2-5n_B+n_A(4n_B-1)]}{2(n_A-1)(2n_An_B-n_A-n_B)} - \sqrt{\frac{n_A\left[(n_A-2)^2n_A+2n_A(5n_A-6)n_B+(8-7n_A)n_B^2\right]}{4(n_A-1)^2(n_A+n_B-2n_An_B)^2}}$, $p_C = \frac{n_An_B}{2n_An_B-n_A-n_B}$, $p_X = \frac{n_A^2n_B-n_An_B}{n_A+n_A^2n_B-n_B-n_An_B}$, and $\alpha_1 = \frac{n_B\left[-2(n_A-1)n_An_B+n_A(2-5n_B+n_A(4n_B-1))p-(n_A-1)(n_A(2n_B-1)-n_B)p^2\right]}{(n_A-n_B)(n_B-1)p^2}$, $\alpha_2 = \frac{n_An_B+n_Ap+n_Bp-2n_An_Bp}{n_Ap-n_An_Bp}$, $\tilde{\alpha}_B = \frac{n_B[n_A(1-p)+3p-2]-2p}{(n_B-1)p}$. It can be verified that $p_C < p_A < p_X < \mathring{p}$ for any $n_A \geq n_B + 1$. To obtain the previous best response one needs to follow the following steps:

- *Step 1*: $\alpha_{A1} \leq 1 \iff p_A \leq p \leq \mathring{p}$ and $\alpha_B \leq \alpha_1$
- *Step 2*: $\alpha_{A2} \leq 1 \iff p_C \leq p$ and $\alpha_B \leq \alpha_2$

  Steps 1 and 2 imply that for $p \leq p_A$

$$\alpha_A(\alpha_B) = \begin{cases} \alpha_{A2}(\alpha_B) & \text{if } p_C < p \leq p_A \text{ and } \alpha_B < \alpha_2 \\ 1 & \text{if } p \leq p_C \text{ or if } p_C < p \leq p_A \text{ and } \alpha_B \geq \alpha_2 \end{cases}$$

- *Step 3*: $\alpha_B > \tilde{\alpha}_B$ is a necessary condition for $\alpha_{A1}(\alpha_B) < \alpha_{A2}(\alpha_B)$
- *Step 4*: If $p \leq p_X$, then $\alpha_1 < \alpha_2 < \tilde{\alpha}_B$

  $\alpha_1 < \tilde{\alpha}_B$ implies that the necessary condition for $\alpha_{A1}(\alpha_B) < \alpha_{A2}(\alpha_B)$ is not compatible with $\alpha_{A1}(\alpha_B) \leq 1$, and thus $\alpha_{A1}(\alpha_B)$ is not in the best response for $p_A < p \leq p_X$. Consequently, for $p_A < p \leq p_X$

$$\alpha_A(\alpha_B) = \begin{cases} \alpha_{A2}(\alpha_B) & \text{if } \alpha_B < \alpha_2 \\ 1 & \text{if } \alpha_B \geq \alpha_2 \end{cases}$$

- *Step 5*: If $p > p_X$ then $\alpha_B > \tilde{\alpha}_B$ is a necessary and sufficient condition for $\alpha_{A1}(\alpha_B) < \alpha_{A2}(\alpha_B)$.

- *Step 6*: If $p > p_X$ then $\tilde{\alpha}_B < \alpha_2 < \alpha_1$

  Consequently, for $p_X < p \leq \mathring{p}$

$$\alpha_A(\alpha_B) = \begin{cases} \alpha_{A2}(\alpha_B) & \text{if } \alpha_B \leq \tilde{\alpha}_B \\ \alpha_{A1}(\alpha_B) & \text{if } \tilde{\alpha}_B < \alpha_B < \alpha_1 \\ 1 & \text{if } \alpha_B \geq \alpha_1 \end{cases}$$

- *Step 7*: If $p > \mathring{p}$

$$\alpha_A(\alpha_B) = \begin{cases} \alpha_{A2}(\alpha_B) & \text{if } \alpha_B \leq \tilde{\alpha}_B \\ \alpha_{A1}(\alpha_B) & \text{if } \tilde{\alpha}_B < \alpha_B \end{cases}$$

By combining all the restrictions on p and α_B from the previous steps, we obtain the best response for group A, $\alpha_A(\alpha_B)$. Note that we obtained the different thresholds in the best response of group A by equalizing the three expressions $\{\alpha_{A1}(\alpha_B), \alpha_{A2}(\alpha_B), 1\}$ which guarantees the continuity of $\alpha_A(\alpha_B)$ in α_B and p.

The expected utility of group B being the leader is then given by

$$EU_B(\alpha_B) = \begin{cases} 0 & \text{if } p \in [p_C, p_X] \text{ and } \alpha_B \leq \alpha_2, \text{ or} \\ & \text{if } p > p_X \text{ and } \alpha_B \leq \tilde{\alpha}_B \\ \hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B) & \text{if } p \in (p_X, \mathring{p}) \text{ and } \tilde{\alpha}_B < \alpha_B \leq \alpha_1, \text{ or} \\ & \text{if } p > \mathring{p} \text{ and } \alpha_B \geq \tilde{\alpha}_B \\ \hat{EU}_B(1, \alpha_B) & \text{otherwise} \end{cases}$$

The continuity of $\alpha_A(\alpha_B)$ guarantees that $EU_B(\alpha_B)$ is also continuous in α_B.

$$\frac{\partial^2 \hat{EU}_B(\alpha_{A1}(\alpha_B),\alpha_B)}{\partial \alpha_B^2} = \frac{[p(n_B-1)]^2 V}{2n_B^2[n_B(p-1)-p]} < 0 \text{ and } \frac{\partial^2 \hat{EU}_B(1,\alpha_B)}{\partial \alpha_B^2} = -\frac{2n_A(n_B-1)^2[n_A(1-p)+p]p^2 V}{n_B[n_B p+n_A(2n_B(1-p)+p)]^2} < 0$$

Thus, $\hat{EU}_B(\alpha_{A1}(\alpha_B), \alpha_B)$ and $\hat{EU}_B(1, \alpha_B)$ are strictly concave. Moreover,

$$\frac{\partial \hat{EU}_B(\alpha_{A1}(\alpha_B),\alpha_B)}{\partial \alpha_B} > 0 \text{ and } \frac{\partial \hat{EU}_B(1,\alpha_B)}{\partial \alpha_B} > 0$$

for all $\alpha_B \leq 1$, which by continuity of $EU_B(\alpha_B)$ guarantees that a global maximum is attained at $\alpha_B = 1$. Substituting $\alpha_B = 1$ in the best response of group A yields

$$\alpha_A = \alpha_{A1}(1) = \frac{n_A^2 n_B[2n_B(1-p)+p](1-p) - n_B p[2n_B(1-p)+p] - n_A\left[2n_B(1-2p)p+p^2+n_B^2(2+p(5p-7))\right]}{2(n_A-1)n_B[n_B(1-p)+p]p}$$

if $p > \mathring{p}$ and $\alpha_A = 1$ otherwise.

□

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